Not Responsive

From: Horner, Melissa EDUC:EX Sent: Monday, July 29, 2013 3:40 PM To: Cormode, Sarah EDUC:EX Subject: RE: math curriculum

Hello Sarah;

Below and attached are some pieces that may be of interest for Aliki's request:

- <u>http://www.cnc.bc.ca/cnc\_programs/program\_websites/community\_education/ethno-</u> <u>math.htm</u> <-link contains a report Catherine McGregor co-wrote about an series of math courses that were created at the College of New Caledonia. This might be the closest fit for Aliki.

- http://cus.oise.utoronto.ca/First\_Nations\_and\_Stereotypes\_in\_Math.html
- http://inquiry.noii.ca/wp-content/uploads/2013/03/ailal-aesn-report-v2.3-web.pdf
- http://www.haidanation.ca/Pages/Haida\_Legends/Lessons/RavenLes4-9.pdf
- http://www.qciobserver.com/Article.aspx?Id=5321
- http://www.fnesc.ca/curriculum/math
- http://showmeyourmath.ca/
- http://mathcatcher.irmacs.sfu.ca/
- http://abed.sd79.bc.ca/acip/indexfiles/math\_lessons\_index.html

Please let Aliki know that I am interested and available for a chat anytime.

Cheers,

Melissa Horner

Curriculum and Assessment Ministry of Education : Ph: 778-679-7596 : Melissa.Horner@gov.bc.ca

Not Responsive

Not Responsive

 From: Arnold Toutant
 S22

 Sent: Tuesday, April 23, 2013 10:02 AM

 To: Poeschek, Nick EDUC:EX; Collins, Valerie A EDUC:EX; Calleberg, Angie EDUC:EX; Beddouche, Linda

 EDUC:EX; Colgate, Elaine EDUC:EX; De Oliveira, Liliane EDUC:EX; Laplante, Chantale EDUC:EX; Hill,

 Stephen M EDUC:EX

 Cc: Walt, Nancy J EDUC:EX; Munro, Brent D EDUC:EX

 Subject: Interesting site

Hi All,

I found this site interesting - particularly Appendix A. In particular, the Interdisciplinary concepts column on a number of the charts illustrates something we know, but have not articulated to this extent. As well, the charts illustrate connections between concepts and enduring understandings. Check it out.

The section "Components of Concept-Centred Learning" provides an interesting interpretation of Lynn Erickson's and Wiggins and McTighe's work.

http://www2.k12albemarle.org/acps/division/fql/Pages/Appendix-A--Discipline-Level-Enduring-Understandings,-Concepts,-and-Habits-of-Mind.aspx

Regards,

Arnold

S22

From:	Walt, Nancy J EDUC:EX		
CC:	Li, Jiemei EDUC:EX; Baer, Markus EDUC:EX; Poescnek, Nick EDUC:EX; Munro, Brent D EDUC:EX; Calleberg,		
	Angle EDUC:EX; Collins, Valerie A EDUC:EX; Beddouche, Linda EDUC:EX; De Oliveira, Liliane EDUC:EX; Horner,		
	Melissa EDUC:EX; Colgate, Elaine EDUC:EX; Hill, Stephen M EDUC:EX; Cormode, Sarah EDUC:EX; Goodman,		
	Dean EDUC:EX; Powell, Janet EDUC:EX		
Subject:	Exploring Curriculum Design and Defining Cross-Curricular Competencies		
Date:	Thursday, January 31, 2013 11:32:05 AM		
Attachments:	Exploring Curriculum Design.pdf Defining Cross Curricular Competencies.pdf		

Dear Colleagues. Thank you so much for participating in our summer and fall curriculum design exploratory sessions. There has been much discussion and evolution of the curricular prototypes over the winter. We are now gearing up for full K-10 curriculum development in all subject areas.

I am attaching two documents that have just been sent to education partner groups:

- 1) Exploring Curriculum Design: This document outlines the exploratory design work and provides the structure that has emerged from consultations over the past year.
- 2) Cross-Curricular Competencies: This document is in draft but reflects the research to date on defining the competencies.

These documents will also be posted on this website in the coming days: <u>http://www.bced.gov.bc.ca/irp/transforming\_curriculum.php</u>

Thank you again for your contributions to this very important work. We look forward to our future connections with you and hope you continue to be champions of this transformation effort!

Sincerely,

Nancy Walt, Director Curriculum and Assessment Learning Division Ministry of Education PO Box 9183 Stn Prov Govt Victoria BC V8W 9H1 Phone: 250-217-4978 Fax: 250-356-8334

http://www.bced.gov.bc.ca/irp/welcome.php http://www.bced.gov.bc.ca/assessment/

Grade 2	Grade 4	Grade 9	Draft 1	Draft 2
Mathematical fluency is developed by using many ways to solve problems, represent numbers, understand our number system, and use mental math strategies.	Discovery & Investigation (Inquiry) Students use visualization skills to assist in processing and applying information. Students are critical thinkers as they strategize/ reason through problem solving.	Numbers can be represented in many forms and each may emphasize something different about that number or relationship	Numeracy helps people to solve problems, represent numbers, understand our number system and use mental math strategies.	Numeracy helps people represent and solve problems.
Investigating and describing patterns helps students to see relationships and develop generalizations.	<b>Communication</b> Students identify effective and efficient ways to communicate thinking in a variety of ways for a variety of purpose.	Linear relations, in their many forms, describe a multiplicative relationship.	Patterns help people see relationships and develop generalizations.	Patterns allow people to see relationships and develop generalizations.
Visualization and an understanding of geometry and measurement help us make meaning of and communicate about the world around us.	Connections Students are engaged and curious about mathematics and are active participants in their learning as they develop a curiosity and appreciation for the beauty of math. They develop personal strategies to become mathematically literate.	Relationships exist within and between shapes and objects.	Visualization and geometry help people to make meaning of the world.	Geometry and measurement empower people to make meaning of the world.
Mathematics can be applied in authentic ways to investigate inquiry questions and communicate information and data.	Fluency & Flexibility Students develop mathematical fluency and flexibility as they solve problems, manipulate and represent numbers, and use mental math strategies.		Mathematics can be applied to inquiry questions and to communicate information and data.	Data enables us to draw conclusions and make predictions.



## Do the Math!



affective domain (growth in feelings or Teaching math should involve: emotional areasattitude) the cognitive domain (mental skillsknowledge) the psychomotor (manual or physical skills-skills)

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Fibers continue to grow between neurons and The growing neural networks of connected sources and fibers are essential to the transmission of information throughout the Vear A As the brain matures, more and more fibers grow and the brain becomes increasingly interconnected. A These interconnected networks of neurons are Very important to the formation of memories and the connection of new learning to previous Primary learning. A As neural networks form, the child learns both At first, this learning is mostly **rote** in nature. As skills become more automatic, the child As skins become more automatic, me chind does not have to think as hard about what he or she is learning or doing, and brain or she is learning or doing, and brain resources are freed up to be used for complex tasks that require more and more attention and processing.

inferential thinking becomes more Middle School Years emphasized rote learning is dethis shift in focus is supported by the increased connectivity in the brain and by chemical changes in the neuronal pathways that support both short and long term memory

An ability to generalize Years and abstract begins at this stage and continues into adulthood. Primarv During this time, the child learns about perspective-taking and social interaction.

Brain activity is mostly in the Posterior regions where the areas for auditory, visual and chool tactile functioning intersect. This intersection is called the association area of the brain and generally contains ementar information that has been ろ Middle learned and is now stored. The frontal lobes begin to mature more fully in middle This continues through high school and adulthood.

The refined development of the frontal white matter tracts begins around age 0-00 12 and continues into the twenties. grades this region of the brain is crucial for higher cognitive functions Appropriate social behaviors, and the development of formal in operations continue to form



Common professional Pooc Peachers development that shows the range of learning K-8 Common planning and implementation time Necessary resources Commitment to ongoing professional development

Designing Systems for Student Success					
Ideal					
Intensive Individual 1-5%	1-5% Intensive Individual				
Structures/Strategies	Structures/Strategies				
Targeted Group 5-15%	5-15% Targeted Group				
Structures/Strategies	Structures/Strategies				
Universal 80-90%	80-90% Universal				
Structures/Strategies	Structures/Strategies				
Academic	Social/Behavioural				

## Designing Systems for Student Success Future

1-20%

Intensive Individual Structures/Strategies 1-20%

Intensive Individual Structures/Strategies

Targeted Group20-50%Structures/Strategies

Universal 40-50% Structures/Strategies

Academic

20-50% Targeted Group Structures/Strategies

> 40-50% Universal Structures/Strategies

Social/Behavioural

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# Change practice to improve student achievement:

1. Shift priorities to spend some time daily or weekly studying teaching practices; focus on planning lessons and then reflecting on their effectiveness.

James Hiebert & James W. Stigler NSDC Fall 2004, Vol. 25, Nov. 4

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2. Provide teachers with vivid examples of alternative teaching methods. James Hiebert & James W. Stigler NSDC Fall 2004, Vol. 25, Nov. 4

3. Have teachers learn to analyze students' work and understand their thinking to see how to adjust and improve their teaching methods. James Hiebert & James W. Stigler NSDC Fall 2004, Vol. 25, Nov. 4



### **Cross-Jurisdictional Summary**

Area of Learning: Mathematics

Jurisdiction, year of publication and url: Saskatchewan Kindergarten, 2010, <u>https://www.edonline.sk.ca/bbcswebdav/library/curricula/English/Master K Curr 2010 Final.pdf</u> Grades 1-9, 2007 The print version is available for each grade, through the Curriculum website: <u>https://www.edonline.sk.ca/webapps/moe-curriculum-BBLEARN/index.jsp</u>

Grade level (s) reviewed: Kindergarten, 1-9

### Intentions and Rationale

The K-12 aim of Saskatchewan's mathematics program is to help students develop the understandings and abilities necessary to be confident and competent in thinking and working mathematically in their daily activities and ongoing learnings and work experiences. The mathematics program is intended to stimulate the spirit of inquiry within the context of mathematical thinking and reasoning.

The four K-12 goals for mathematics in Saskatchewan are broad statements that identify the characteristics of thinking and working mathematically. At every grade level, students' learning should be building towards their attainment of these goals. Within each grade level, outcomes are directly related to the development of one or more of these goals. The goals in boldface font in the list of outcomes indicate a stronger connection to the related outcome. The instructional approaches used to promote student achievement of the grade level outcomes will, therefore, also promote student achievement with respect to the goals.

The four broad goals are: 1) Logical Thinking; 2) Number Sense; 3) Spatial Sense; and 4) Mathematics as a Human Endeavor (\*\*Sometimes referred to in the documents as 'Mathematical Attitude' – see Math 7 [3])

(Kindergarten [44], Grades 1-9 [2-5])

### Kindergarten:

Kindergarten mathematics provides children with an understanding of quantity, shape, and space in their environment. Children require opportunities for reflection, exploration of patterns and relationships, sharing ideas and problems, and decision making. Opportunities for children to communicate through representing will be part of the learning experiences in Kindergarten. Kindergarten children will have opportunities to achieve the mathematics outcomes through play and inquiry (45).

### Grades 1-9:

For each grade related curriculum document, there is a brief explanation of the context. For example, for mathematics 8:

This curriculum has been designed to address current research in mathematics education as well as the needs of students. The Mathematics 7 Curriculum outcomes have been influenced by changes to the outcomes in K-6 mathematics, and will also impact the content of 8-12 mathematics. Changes throughout all of the grades have been made for a number of reasons including:

• decreasing content in each grade to allow for more depth of understanding

• rearranging concepts to allow for greater depth of learning in one year and to align related mathematical concepts

• increasing the focus on numeracy (understanding numbers) beginning in Kindergarten

• introducing algebraic thinking earlier.

(1)

### Structure:

In total for Saskatchewan, there are three Mathematics curriculum documents; there does not appear to be a rationale for the different documents.

The Kindergarten curriculum is one document, which encompasses program expectations (curriculum organizers and learning standards) for seven learning areas: Arts Education, English Language Arts, Health Education, Mathematics, Physical Education, Science, and Social Studies. There is then one document for Grades 1-9 and another for Grades 10-12. It can be noted that there is an additional document for Math A30, B30, and C30, which is a set of Calculus courses.

- 1) Kindergarten one pathway
- 2) Grades 1-9 one pathway

### **Curriculum Organizers**

In addition to the expectations outlined within each strand, a list of seven "Critical Characteristics of Mathematical Education" is provided.

The Critical Characteristics of Mathematical Education are: communication; connections; mental mathematics and estimations; problem solving; reasoning; visualization; and technology. Teachers need to be aware of these characteristics in order to provide an effective program (Kindergarten [46]; Grades 1-9 [11-14]). These characteristics are also connected to the learning standards.

It is interesting to note that for the Kindergarten document, the Critical Characteristics of Mathematical Education are listed without an explanation of each characteristic, although this is provided for the 1-9 curriculum.

K-12 Curriculum Organizers:

- Number
- Patterns and Relations
- Shape and Space
- Statistics and Probability (although it is stated as a K-12 strand, it is not included in all grades; for example, Kindergarten and grade 1)

### Learning Outcomes (Learning Standards)

Under each curriculum organizer are the outcomes, with achievement indicators. The language of the outcomes and indicators is low, and written from the perspective of the student. The outcomes are labeled as such: NK.1, where N represents the curriculum organizer, Number, K the grade level, Kindergarten, and the decimal number, the outcome number.

Indicators are included for each of the outcomes in order to clarify the breadth and depth of learning intended by the outcome. These indicators are a representative list of the kinds of things a student needs to know and/or be able to do in order to achieve the learnings intended by the outcome.

### Kindergarten:

The outcomes are statements of what students are expected to know, understand, and be able to do by the end of a grade in a particular area of study. The outcomes provide direction for assessment and evaluation, and for program, unit, and lesson planning (21).

For Kindergarten, there are nine outcomes.

### Grades 1-9:

For each grade document, there is a related statement of: The Grades 1-9 Mathematics Curriculum defines the outcomes to be attained by students in \_\_\_\_\_ grade during the 2\_\_\_\_\_ minutes of instruction and learning time allocated per week for the entire school year (iii).

For grade 7, there are 18 outcomes.

### Learning Contexts

### Kindergarten:

An effective Kindergarten program:

- 1) Incorporates the Principles of Early Learning;
- 2) Actualizes the distinguishing features of an Early Childhood Education program;
- 3) Uses a reflective lens to support student learning;
- 4) Draws upon the processes of observation, documentation, and Interpretation;
- 5) Facilitates inquiry through play; and
- 6) Reflects the philosophy of each subject area discipline.

### (4-9)

### Grades 1-9:

For deep understanding, it is vital that mathematics be taught through the students uncovering and co-constructing their knowledge, with very few ideas being covered or relayed directly by the teacher. Teachers need to "unpack" outcomes to identify those things that students need to know (behold) and those that they need to be able to apply, explain, and transfer to new situations. It is important that a mathematics learning environment include effective interplay of reflection, exploration of patterns and relationships, sharing of ideas and problems, consideration of different perspectives, decision making, generalizing, verifying and proving, and modeling and representing. Mathematics is truly learned when students are engaged in strategic "play" with mathematical concepts and differing perspectives. When students learn mathematics by being told what to do, how to do it, and when to do it, they cannot make the strong learning connections necessary for learning to be meaningful, easily accessible, and transferable. The mathematics learning environment must necessarily be one which is respectful of individuals and groups. It needs to foster discussion and self-reflection, the asking of questions, the seeking of multiple answers, and the co-construction of understanding.

(15)

### Competencies

### Kindergarten:

The broad areas of learning: 1) building a disposition for learning; 2) building a sense of self and community; and 3) building engaged citizens and the Cross-curricular competencies: 1) constructing knowledge; 2) identity and interdependence; 3) developing literacy; and 4) social responsibility are described in how they relate/connect to students in Kindergarten (2-3).

### Grades 1-9:

In addition to connecting the broad areas of learning and the cross-curricular competencies (as stated above), for grades 1-9, cross-curricular connections between mathematics and the other subjects are also provided.

For example, how mathematics 8 connects to Arts Education (9).

### Strengths and Limitations

A strength to this curriculum are the connections made to its: broad learning goals, crosscurricular competencies, and cross-curricular connections to other courses.

A limitation is in its inconsistencies; for example, using Mathematical Attitudes instead of Mathematics as a Human Endeavor. Also, there are inconsistencies between the print and online versions of the curriculum.

### **Cross-Jurisdictional Summary**

Area of Learning: Mathematics

Jurisdiction, year of publication and url: Saskatchewan, 2010

The print version is available for each grade, through the Curriculum website: <u>https://www.edonline.sk.ca/webapps/moe-curriculum-BBLEARN/index.jsp</u>

Grade level (s) reviewed:10-12

\*Mentioned - Calculus courses: A30, B30, C30 (1996) https://www.edonline.sk.ca/bbcswebdav/library/curricula/English/Mathematics/Mathematics A30 B30 C30 1996.pdf

### Intentions and Rationale

The K-12 aim of the mathematics program is to have students develop the understandings and abilities necessary to be confident and competent in thinking and working mathematically in their daily activities, ongoing learning, and work experiences. The K-12 mathematics program is intended to stimulate the spirit of inquiry within the context of mathematical thinking and reasoning.

The four K-12 goals for mathematics in Saskatchewan. The goals are broad statements that identify the characteristics of thinking and working mathematically. At every grade level, students' learning should be building towards their attainment of these goals. Within each grade level, outcomes are directly related to the development of one or more of these goals. The instructional approaches used to promote student achievement of the grade level outcomes must, therefore, also promote student achievement with respect to the K-12 goals.

The four broad goals are: 1) Logical Thinking; 2) Number Sense; 3) Spatial Sense; and 4) Mathematics as a Human Endeavor (\*\*Sometimes referred to in the documents as 'Mathematical Attitude' – see Math 7 [3]) (8-11)

### Structure:

Original WNCP Framework:

The main structure of SK's mathematics 10-12 curriculum comes directly from the Western and Northern Canadian Protocol (WNCP) Common Curriculum Framework for Grades 10-12 Mathematics.

For the WNCP's grades 10-12 mathematics, the topics are split into three different pathways: Apprentice and Workplace; Foundations of Mathematics; and Pre-Calculus. Each of these pathways have their own curricular organizers, general outcomes, specific outcomes, and achievement indicators. These courses are guided by: 1) Nature of Mathematics: change, constancy, number sense, patterns, relationships, spatial sense, uncertainty; and, 2) Mathematical Processes: communication; connections, mental mathematics and estimation; problem solving; reasoning; technology; and visualization (5).

Note: For the 10 level, there are only two options: Apprentice and Workplace 10 or Foundations of Mathematics and Pre-Calculus 10. The Foundations of Mathematics and Pre-Calculus courses then split at the 11 and 12 levels.

### SK Changes:

In terms of curriculum organizers, the document states that: the content of K-12 mathematics can be organized in a variety of ways. In the grades 10-12 curricula, the outcomes are not grouped according to strands (as in the elementary mathematics curricula) or by topic (as in past curricula). The primary reasons for this are: a succinct set of high level outcomes for each grade, and variation between grades and pathways in terms of the topics and content within different courses (14).

While Mathematical Processes are included and attached to specific outcomes, there is no reference to the Nature of Mathematics.

Outcomes are based on, but do not follow exactly, the WNCP; some outcomes have been altered or deleted (for Math 10 FNDPC – difference of 8 specific outcomes)

Whereas the WNCP names the courses 10, 11, 12, SK names them as 10, 20, 30.

SK also has a separate stream of Calculus courses, designated as A30, B30, C30.

### Learning Outcomes (Learning Standards)

The outcomes in this curriculum define content that is considered a high priority in fields of study and areas of work for which the pathway is required. The outcomes represent the ways of thinking or behaving like a mathematics discipline area expert in those fields of study or areas of work.

Indicators are included for each of the outcomes in order to clarify the breadth and depth of learning intended by the outcome. These indicators are a representative list of the kinds of things a student needs to understand and/or be able to do in order to achieve the learning intended by the outcome. New and combined indicators, which remain within the breadth and depth of the outcome, can be created by teachers to meet the needs and circumstances of their students and communities.

This curriculum's outcomes and indicators have been designed to address current research in mathematics education as well as the needs of Saskatchewan students. Within the outcomes and indicators in this curriculum, the terms "including" and "such as", as well as the abbreviation "e.g.," occur. The use of each term serves a specific purpose. The term "including" prescribes content, contexts, or strategies that students must experience in their learning, without excluding other possibilities.

(1)

For K-12 (overall):

- Strands (Curriculum organizers): number; patterns and relations; shape and space; statistics and probability (for all grades, although some grades do not have learning outcomes)
- Mathematical processes: communication; connections; mental mathematics and estimation; problem solving; reasoning; technology; and visualization
- Nature of mathematics: not included or mentioned

For 10-12:

- Outcomes are based on, but do not follow exactly, the WNCP; some outcomes have been altered or deleted (for Math 10 FNDPC difference of 8 specific outcomes)
- Even though the K-12 (overall) document states the strands (curriculum organizers), they are not used for the 10-12. Outcomes are connected to the broad goals (on the print version), then stated and numbered.

Additional courses: A30, B30, C30 (1996) \*Calculus courses

### Learning Contexts

The curriculum states that: Mathematics is a dynamic and logic-based language that students need to explore and make sense of for themselves. For many teachers, parents, and former students, this is a marked change from the way mathematics was taught to them. Research and experience indicate there is a complex, interrelated set of characteristics that teachers need to be aware of in order to provide an effective mathematics program (12).

The characteristics are then described in more details; these are categorized as:

- Assumptions in this Curriculum
- Critical Characteristics of Mathematical Education
- Teaching for Deeper Understanding
- Inquiry

(12-27)

### **Competencies**

The broad areas of learning: 1) building a disposition for learning; 2) building a sense of self and community; and 3) building engaged citizens and the Cross-curricular competencies: 1) constructing knowledge; 2) identity and interdependence; 3) developing literacy; and 4) social responsibility are described in how they relate/connect to students in grades 10-12 (5-7).

### Strengths and Limitations

A strength in the curriculum are the connections provided to its overriding broad areas of learning and to its competencies.

A limitation is the differences between the print and online versions of the curriculum documents; there are some minor differences/changes. Also, the discrepancy between the K-12 curriculum strands and then the lack thereof in the 10-12 curriculum.

### **Cross-Jurisdictional Summary**

Area of Learning: Mathematics

Jurisdiction, year of publication and url: Ontario: Kindergarten, 2011, http://www.edu.gov.on.ca/eng/curriculum/elementary/kindergarten\_english\_june3.pdf Grades 1-8, 2005, http://www.edu.gov.on.ca/eng/curriculum/elementary/math18curr.pdf

Grade level (s) reviewed: Kindergarten, 1-8

### Intentions and Rationale

### Kindergarten:

Mathematics in the Full-Day Early Learning–Kindergarten program builds on children's desire to make sense of their world, and helps them develop and demonstrate their mathematical understanding. Young children use mathematics intuitively and develop their understanding of mathematics through their individual approaches to learning, as well as through their prior experience of their linguistic, family, cultural, and community backgrounds (92).

### Grades 1-8:

The study of mathematics equips students with knowledge, skills, and habits of mind that are essential for successful and rewarding participation in such a society. To learn mathematics in a way that will serve them well throughout their lives, students need classroom experiences that help them develop mathematical understanding; learn important facts, skills, and procedures; develop the ability to apply the processes of mathematics; and acquire a positive attitude towards mathematics. Learning mathematics results in more than a mastery of basic skills. It equips students with a concise and powerful means of communication. Mathematical structures, operations, processes, and language provide students with a framework and tools for reasoning, justifying conclusions, and expressing ideas clearly. Through mathematical activities that are practical and relevant to their lives, students develop mathematical understanding, problemsolving skills, and related technological skills that they can apply in their daily lives and, eventually, in the workplace. Mathematics is a powerful learning tool. As students identify relationships between mathematical concepts and everyday situations and make connections between mathematical other subjects, they develop the ability to use mathematics to extend and apply their knowledge in other curriculum areas, including science, music, and language (3).

### Structure:

In total, there are five different K-12 Mathematics curriculum documents; there does not appear to be a rationale for the five separate documents.

The Kindergarten curriculum is one document, which encompasses program expectations (curriculum organizers and learning standards) for six learning areas: Personal and Social Development; Language; Mathematics; Science and Technology; Health and Physical Activity; and The Arts. There is then one document for: Grades 1-8, Grades 9 and 10; Grade 9 – Mathematics Transfer course – Applied to Academic; and Grades 11 and 12.

- 1) Kindergarten one pathway
- 2) 1-8 (2005) one pathway

### **Curriculum Organizers**

In addition to the expectations outlined within each strand, a list of seven "mathematical process expectations" precedes the strands in each grade (7).

The mathematics process expectations are: problem solving, reasoning and proving, reflecting, selecting tools and computational strategies, connecting, representing, and communicating. These processes are to be woven through the teaching and learning of the learning outcomes. (Kindergarten [94]; Grades 1-8 [4]).

For Kindergarten:

• Number Sense and Numeration (quantity relationships; counting;

operational sense)

• Measurement (attributes, units, and measurement sense; measurement relationships)

- Geometry and Spatial Sense (geometric properties; geometric relationships; location and movement)
- Patterning (patterns and relationships)
- Data Management and Probability (collection and organization of data;
- data relationships; probability)

To ensure continuity with the mathematics curriculum for Grades 1 to 8, the above categories are largely the same as the strand titles in The Ontario Curriculum, Grades 1–8: Mathematics, 2005. The text in parentheses, which indicates the subtopics in each category, is also largely the same as the headings for the subgroupings of the specific expectations for Grades 1 to 8 (the only differences are in Patterning) (92).

For Grades 1-8:

Two sets of expectations are listed for each grade in each strand, or broad curriculum area, of mathematics:

• The overall expectations describe in general terms the knowledge and skills that students are expected to demonstrate by the end of each grade.

• The specific expectations describe the expected knowledge and skills in greater detail. (7)

The five strands are: Number Sense and Numeration; Measurement; Geometry and Spatial Sense; Patterning and Algebra; and Data Management and Probability (8).
## Learning Outcomes (Learning Standards)

Two sets of expectations are listed for each grade in each strand, or broad curriculum area, of mathematics:

• The overall expectations (curriculum organizers) describe in general terms the knowledge and skills that students are expected to demonstrate by the end of each grade.

• The specific expectations (learning standards) describe the expected knowledge and skills in greater detail. The specific expectations are grouped under subheadings that reflect particular aspects of the required knowledge and skills and that may serve as a guide for teachers as they plan learning activities for their students. (These groupings often reflect the "big ideas" of mathematics that are addressed in the strand.) The organization of expectations in subgroups is not meant to imply that the expectations in any one group are achieved independently of the expectations in the other groups. The subheadings are used merely to help teachers focus on particular aspects of knowledge and skills as they develop and present various lessons and learning activities for their students.

(Grades 1-8 [7])

The overall and specific goals are 'expected,' but it does not specify whether or not these outcomes are prescribed, authorized, or suggested. The wording is simple, but technical – using mathematics specific terminology (Kindergarten [97-111]; Grades 1-8 [7-8]).

#### Kindergarten:

There is one Big Idea, used as a heading for all of the Overall Expectations: Young children have a conceptual understanding of mathematics and of mathematical thinking and reasoning (97).

There are then five Overall Expectations, one for each curriculum organizer, which have their own related Specific Expectations. They are categorized by an abbreviation derived from the corresponding organizer, a whole number, and then a decimal. For example: NS1.2 is the first Overall Expectation (curriculum organizer), related to Number Sense, and is the second of the Specific Expectations (learning outcomes).

For Kindergarten, there are 27 specific expectations.

#### Grades 1-8:

For each grade, there are expectations stated for each mathematical process; for example, Grade 1, on page 32. For each curriculum organizer, a number of Overall Expectations are stated, followed by subsequent Specific Expectations headers, followed by learning outcomes. These are not numbered or organized in any significant way.

For Grade 6, there are 61 specific expectations.

## Learning Contexts

#### Kindergarten:

This curriculum emphasis that mathematics in the early years must be active, hands-on, childcentred, and problem-based. Concrete materials provide children with tactile experiences to help them explore and describe mathematical problems and solutions. Questioning is a very important aspect of mathematics in the Full-Day Early Learning-Kindergarten program. EL-K teams should provide models of a range of question types to promote problem solving and to probe and challenge children's mathematical thinking and reasoning. These teams should also create an environment in which children are encouraged to pose mathematical questions, explore, and investigate. It is important that good questioning be interwoven throughout the Full-Day Early Learning-Kindergarten program and that children's literature, music, or art work be used as starting points for mathematics activities. Reading books aloud and in shared reading contexts provides real links between literature and mathematical ideas, since some stories use mathematical terminology and/or contain illustrations of mathematical concepts. Reading can also give children a sense of how mathematics is connected with other aspects of life, such as science and the arts. Children should therefore be given many opportunities to demonstrate their understanding in a variety of ways – for example, by constructing concrete models, by describing their understanding in their first language, and/or by making drawings to illustrate a mathematical concept. EL-K teams should provide children with planned opportunities every day to develop their mathematical understanding by incorporating high-quality investigative learning experiences that involve the use of mathematics manipulatives. These teams can introduce mathematical concepts, strategies, and vocabulary in carefully planned hands-on activities at various learning centres in the classroom

#### Grades 1-8:

This curriculum is based on the belief that all students can learn mathematics and deserve the opportunity to do so. It recognizes that all students do not necessarily learn mathematics in the same way, using the same resources, and within the same time frames. It supports equity by promoting the active participation of all students and by clearly identifying the knowledge and skills students are expected to demonstrate in every grade. It recognizes different learning styles and sets expectations that call for the use of a variety of instructional and assessment tools and strategies. It aims to challenge all students by including expectations that require them to use higher-order thinking skills and to make connections between related mathematical concepts and between mathematics, other disciplines, and the real world (3).

#### **Competencies**

In the front matter of the mathematics curriculum are a set of descriptions of the Roles and Responsibilities for: students, parents, teachers, and principals. Within the descriptions, there is information regarding how these roles and responsibilities apply to not only mathematics, but to the overall student learning (Kindergarten [6-13]; Grades 1-8 [4-6]).

## Strengths and Limitations

The approach to mathematics in Kindergarten, emphasizing that it be interwoven with the other learning areas, provides a strong cross-curricular foundation.

The lack of organization, numbering or otherwise, in the Grades 1-8 learning standards is a limitation, as it can inhibit an understanding of how many specific expectations there are per grade.

# **Cross-Jurisdictional Summary**

Area of Learning: Mathematics

Jurisdiction, year of publication and url: Ontario: Grades 9 and 10, 2005, http://www.edu.gov.on.ca/eng/curriculum/secondary/math910curr.pdf Grade 9 – Mathematics Transfer Course, Applied to Academic, 2006, http://www.edu.gov.on.ca/eng/curriculum/secondary/matht9curr.pdf Grades 11 and 12, 2007, http://www.edu.gov.on.ca/eng/curriculum/secondary/math1112currb.pdf

Grade level (s) reviewed: 9-12

# Intentions and Rationale

## Grades 9 and 10:

The Ontario grades 9 and 10 mathematics curriculum states that it must prepare students for their future roles in society. It must equip them with essential mathematical knowledge and skills; with skills of reasoning, problem solving, and communication; and, most importantly, with the ability and the incentive to continue learning on their own. This curriculum provides a framework for accomplishing these goals. The choice of specific concepts and skills to be taught must take into consideration new applications and new ways of doing mathematics. The development of sophisticated yet easy-to-use calculators and computers is changing the role of procedure and technique in mathematics. Operations that were an essential part of a procedures-focused curriculum for decades can now be accomplished quickly and effectively using technology, so that students can now solve problems that were previously too time-consuming to attempt, and can focus on underlying concepts. This curriculum integrates appropriate technologies into the learning and doing of mathematics, while recognizing the continuing importance of students' mastering essential numeric and algebraic skills. Mathematical knowledge becomes meaningful and powerful in application. This curriculum embeds the learning of mathematics in the solving of problems based on real-life situations.

(3)

## Grades 11 and 12:

The required knowledge and skills include not only important mathematical facts and procedures but also the mathematical concepts students need to understand and the mathematical processes they must learn to apply. The principles underlying this curriculum are shared by educators dedicated to the success of all students in learning mathematics. Those principles can be stated as follows:

- Curriculum expectations must be coherent, focused, and well-articulated across the grades.
- Learning mathematics involves the meaningful acquisition of concepts, skills, and processes and the active involvement of students in building new knowledge from prior knowledge and experience.
- Learning tools such as manipulatives and technologies are important supports for teaching and learning mathematics.

The Ontario mathematics curriculum states that it must serve a number of purposes. It must engage all students in mathematics and equip them to thrive in a society where mathematics is increasingly relevant in the workplace. It must engage and motivate as broad a group of students as possible, because early abandonment of the study of mathematics cuts students off from many career paths and postsecondary options. Today's mathematics curriculum must prepare students for their future roles in society. It must equip them with an understanding of important mathematical ideas; essential mathematical knowledge and skills; skills of reasoning, problem solving, and communication; and, most importantly, the ability and the incentive to continue learning on their own.

The development of mathematical knowledge is a gradual process. A coherent and continuous program is necessary to help students see the "big pictures", or underlying principles, of mathematics. The fundamentals of important skills, concepts, processes, and attitudes are initiated in the primary grades and fostered throughout elementary school. By studying mathematics, students learn how to reason logically, think critically, and solve problems – key skills for success in today's workplaces. Mathematical knowledge becomes meaningful and powerful in application. This curriculum embeds the learning of mathematics in the solving of problems based on real-life situations. Other disciplines are a ready source of effective contexts for the study of mathematics.

(3-4)

#### Structure:

In total, there are five different K-12 Mathematics curriculum documents; there does not appear to be a rationale for the five separate documents.

The Kindergarten curriculum is one document, which encompasses program expectations (learning standards) for six learning areas: Personal and Social Development; Language; Mathematics; Science and Technology; Health and Physical Activity; and The Arts. There is then one document for: Grades 1-8, Grades 9 and 10; Grade 9 – Mathematics Transfer course – Applied to Academic; and Grades 11 and 12.

The mathematics courses in the Grade 9 and 10 curriculum are offered in two types, academic and applied, which are defined as follows:

*Academic courses* develop students' knowledge and skills through the study of theory and abstract problems. These courses focus on the essential concepts of a subject and explore related concepts as well. They incorporate practical applications as appropriate.

*Applied courses* focus on the essential concepts of a subject, and develop students' knowledge and skills through practical applications and concrete examples. Familiar situations are used to illustrate ideas, and students are given more opportunities to experience hands-on applications of the concepts and theories they study.

Students who successfully complete the Grade 9 academic course may proceed to either the Grade 10 academic or the Grade 10 applied course. Those who successfully complete the Grade 9 applied course may proceed to the Grade 10 applied course, but must successfully complete a transfer course if they wish to proceed to the Grade 10 academic course. The Grade 10 academic and applied courses prepare students for particular destination-related courses in Grade 11. The Grade 11 and 12 mathematics curriculum offers university preparation, university/college preparation, college preparation, and workplace preparation courses. When choosing courses in Grades 9 and 10, students, parents, and educators should carefully consider students' strengths, interests, and needs, as well as their postsecondary goals and the course pathways that will enable them to reach those goals.

(Grades 9 and 10 [6-7])

For grades 11 and 12, four types of courses are offered in the senior mathematics program: university preparation, university/college preparation, college preparation, and workplace preparation. Students choose course types on the basis of their interests, achievement, and postsecondary goals.

The course types are defined as follows:

*University preparation courses* are designed to equip students with the knowledge and skills they need to meet the entrance requirements for university programs.

*University/college preparation courses* are designed to equip students with the knowledge and skills they need to meet the entrance requirements for specific programs offered at universities and colleges.

*College preparation courses* are designed to equip students with the knowledge and skills they need to meet the requirements for entrance to most college programs or for admission to specific apprenticeship or other training programs.

*Workplace preparation courses* are designed to equip students with the knowledge and skills they need to meet the expectations of employers, if they plan to enter the workplace directly after graduation, or the requirements for admission to many apprenticeship or other training programs. (8)

## For grades 9 and 10:

The links between Grade 8 and Grade 9 and the transition from elementary school mathematics to secondary school mathematics are very important in the student's development of confidence and competence. The Grade 9 courses in this curriculum build on the knowledge of concepts and skills that students are expected to have by the end of Grade 8. The strands used are similar to

those of the elementary program, with adjustments made to reflect the new directions mathematics takes in secondary school. The Grade 9 courses are based on principles that are consistent with those that underpin the elementary program, facilitating the transition from elementary school. These courses reflect the belief that students learn mathematics effectively when they are initially given opportunities to investigate ideas and concepts and are then guided carefully into an understanding of the abstract mathematics involved. Skill acquisition is an important part of the program; skills are embedded in the contexts offered by various topics in the mathematics program and should be introduced as they are needed. The Grade 9 and 10 mathematics curriculum is designed to foster the development of the knowledge and skills students need to succeed in their subsequent mathematics courses, which will prepare them for the postsecondary destinations of their choosing. (4)

The courses are designed to be offered as full-credit courses. However, they may also be delivered as half-credit courses. Half-credit courses, which require a minimum of fifty-five hours of scheduled instructional time, must adhere to the following conditions:

• The two half-credit courses created from a full course must together contain all of the expectations of the full course. The expectations for each half-credit course must be divided in a manner that best enables students to achieve the required knowledge and skills in the allotted time.

• A course that is a prerequisite for another course in the secondary curriculum may be offered as two half-credit courses, but students must successfully complete both parts of the course to fulfill the prerequisite. (Students are not required to complete both parts unless the course is a prerequisite for another course they wish to take.)

## (7)

## For grades 11 and 12:

The links between Grade 8 and Grade 9 and the transition from elementary school mathematics to secondary school mathematics are very important in developing the student's confidence and competence. The secondary courses are based on principles that are consistent with those that underpin the elementary program, facilitating the transition from elementary school. These courses reflect the belief that students learn mathematics effectively when they are given opportunities to investigate new ideas and concepts, make connections between new learning and prior knowledge, and develop an understanding of the abstract mathematics involved. Skill acquisition is an important part of the learning; skills are embedded in the contexts offered by various topics in the mathematics program and should be introduced as they are needed. The mathematics courses in this curriculum recognize the importance of not only focusing on content, but also of developing the thinking processes that underlie mathematics. (3-4)

Similarly to the grade 9 and 10 courses, the 11 and 12 courses are designed to be offered as fullcredit courses.

However, with the exception of the Grade 12 university preparation courses, they may also be delivered as half-credit courses.

(11)

## Grades 9-12 Pathways:

- 1) Grades 9 and 10 (2005) two pathways: academic and applied
- Grade 9: Mathematics transfer course, Applied to Academic (2006) this allows students to go from Math 9 Applied to Math 10 Academic. Students receive a 0.5 credit for taking this course.
- 3) Grades 11 and 12 (2007)
  i) Grade 11 four pathways: 1) Functions, University Preparation; 2) Functions and Applications, University/College Preparation; 3) Foundations for College Mathematics,

College Preparation; and 4) Mathematics for Work and Everyday Life, Workplace Preparation

ii) Grade 12 – six pathways: 1) Advanced Functions, University Preparation; 2) Calculus and Vectors, University Preparation; 3) Mathematics of Data Management, University Preparation; 4) Mathematics for College Technology, College Preparation; 5) Foundations for College Mathematics, College Preparation; and 4) Mathematics for Work and Everyday Life, Workplace Preparation

## **Curriculum Organizers**

Two sets of expectations are listed for each grade in each strand, or broad curriculum area, of mathematics:

• The overall expectations describe in general terms the knowledge and skills that students are expected to demonstrate by the end of each grade.

• The specific expectations describe the expected knowledge and skills in greater detail.

In addition to the expectations outlined within each strand, a list of seven "mathematical process expectations" precedes the strands in each grade.

The mathematics process expectations are: problem solving, reasoning and proving, reflecting, selecting tools and computational strategies, connecting, representing, and communicating. These processes are to be woven through the teaching and learning of the learning outcomes. (Grades 9 and 10[8]; Grades 11 and 12 [7-8]).

With the exception of the Foundations for College Mathematics courses, the strands (curriculum organizers) are different for each grade and levels, and are as follows:

Principles of Mathematics (Academic)

9: Number Sense and Algebra; Linear Relations; Analytic Geometry; Measurement and Geometry

10: Quadratic Relations of the Form  $y = ax^2 + bx + c$ ; Analytic Geometry; Trigonometry

Foundations of Mathematics (Applied)

9: Number Sense and Algebra; Linear Relations; Measurement and Geometry

10: Measurement and Trigonometry; Modeling Linear Relations; Quadratic Relations of the Form  $y = ax^2 + bx + c$ 

University Preparation Courses

11, Functions: Characteristics of Functions; Exponential Functions; Discrete Functions; and Trigonometric Functions

12, Advanced Functions: Exponential and Logarithmic Functions; Trigonometric Functions; Polynomial and Rational Functions; and Characteristics of Functions

12, Mathematics of Data Management: Counting and Probability; Probability Distributions; Organization of Data for Analysis; Statistical Analysis; and Culminating Data Management Investigation

12, Calculus and Vectors: Rate of Change; Derivatives and Their Applications; and Geometry and Algebra of Vectors

University/College Preparation

11, Functions and Applications: Quadratic Functions; Exponential Functions; and Trigonometric Functions

12, Mathematics for College Technology: Exponential Functions; Polynomial Functions; Trigonometry Functions; and Applications of Geometry

College Preparation Courses

11, Foundation for College Mathematics: Mathematical Models; Personal Finance; Geometry and Trigonometry; and Data Management

12, Foundations for College Management: Mathematical Models; Personal Finance; Geometry and Trigonometry; and Data Management

#### Workplace Preparation Courses

11, Mathematics for Work and Everyday Life: Earning and Purchasing; Saving, Investing, and Borrowing; Transportation and Travel

12, Mathematics for Work and Everyday Life: Reasoning with Data; Personal Finance; and Applications of Measurement

## Learning Outcomes (Learning Standards)

Two sets of expectations are listed for each grade in each strand, or broad curriculum area, of mathematics:

• The overall expectations describe in general terms the knowledge and skills that students are expected to demonstrate by the end of each grade.

• The specific expectations (learning standards) describe the expected knowledge and skills in greater detail. The specific expectations are grouped under subheadings that reflect particular aspects of the required knowledge and skills and that may serve as a guide for teachers as they plan learning activities for their students. (These groupings often reflect the "big ideas" of mathematics that are addressed in the strand.) The organization of expectations in subgroups is not meant to imply that the expectations in any one group are achieved independently of the expectations in the other groups. The subheadings are used merely to help teachers focus on particular aspects of knowledge and skills as they develop and present various lessons and learning activities for their students.

The overall and specific goals are 'expected,' but it does not specify whether or not these outcomes are prescribed, authorized, or suggested. The wording is simple, but technical – using mathematics specific terminology (Grades 9 and 10 [8]; Grades 11 and 12 [11-12]).

#### Math 9 and 10

For each grade, there are expectations stated for each mathematical process; for example, Grade 9, Academic on page 30. For each curriculum organizer, a number of Overall Expectations are stated, followed by subsequent Specific Expectations headers, followed by learning outcomes. These are not numbered or organized in any significant way.

For Math 9, Applied, there are 44 specific expectations (learning standards).

## Math 11 and 12

For each strand (curriculum organizer), there is a set of overall expectations. The following specific expectation are grouped and numbered by sub-organizers. For example, for the course 11, Functions, one of the strands is A. Characteristics of Functions. One of its sub-organizers is 1. Representing Functions. The specific expectation related to this are numbered as 1.1, 1.2, etc.

For 11, Functions and Applications, there are 53 specific expectations (learning standards).

# Learning Contexts

## Math 9 and 10:

These courses reflect the belief that students learn mathematics effectively when they are initially given opportunities to investigate ideas and concepts and are then guided carefully into an understanding of the abstract mathematics involved. Skill acquisition is an important part of the program; skills are embedded in the contexts offered by various topics in the mathematics program and should be introduced as they are needed.

(4)

## Grades 11 and 12:

- Effective teaching of mathematics requires that the teacher understand the mathematical concepts, procedures, and processes that students need to learn, and use a variety of instructional strategies to support meaningful learning.
- Assessment and evaluation must support learning, recognizing that students learn and demonstrate learning in various ways.
- Equity of opportunity for student success in mathematics involves meeting the diverse learning needs of students and promoting excellence for all students. Equity is achieved when curriculum expectations are grade- and destination appropriate, when teaching and learning strategies meet a broad range of student needs, and when a variety of pathways through the mathematics curriculum are made available to students.

(3-4)

## Competencies

In the front matter of the mathematics curriculum are a set of descriptions of the Roles and Responsibilities for: students, parents, teachers, and principals. Within the descriptions, there is information regarding how these roles and responsibilities apply to not only mathematics, but to the overall student learning (9 and 10 [4-5]; 11 and 12 [5-6])

## Strengths and Limitations

A strength that is also a limitation is the number of choices have for their grade 11 and 12 courses; sometimes too many courses can cause as many problems as having too few.

Another limitation is the lack of organization for not only the specific expectations, but also for much of the front matter information.

## **Cross-Jurisdictional Summary**

Area of Learning: Mathematics

Jurisdiction, year of publication and url: Finland, 2003, http://www.oph.fi/download/47678\_core\_curricula\_upper\_secondary\_education.pdf

Grade level (s) reviewed: General Upper Secondary (ages 16-19)

## Intentions and Rationale

According to the General Upper Secondary Schools Act of 1998, the objective of general upper secondary education is to promote the development of students into good, balanced and civilized individuals and members of society and to provide students with the knowledge and skills necessary for further studies, working life, their personal interests and the diverse development of their personalities. In addition, the education must support students' opportunities for lifelong learning and self-development.

## Conception of Learning:

The National Core Curriculum is based on a conception of learning which states that learning is a result of a student's active and focused actions aimed to process and interpret received information in interaction with other students, teachers and the environment and on the basis of his or her existing knowledge structures.

The conception of learning... emphasises students' own active knowledge construction...

At upper secondary school, the aim is to use the curriculum as the basis to create an operational culture that highlights the responsibility of the members of the community as a whole and is open to co-operation and interaction with society and to changes occurring in the world.

## Mathematics:

The role of instruction in mathematics is to acquaint students with the models of mathematical thinking and the basic ideas and structures of mathematics, teach them to use mathematical language both orally and in writing and develop their calculation and problem-solving skills (122).

#### Mathematics, basic syllabus

The role of instruction in the basic mathematics syllabus is to provide students with capabilities to acquire, process and understand mathematical information and to use mathematics in different situations in life and in further studies.

#### **Objectives of instruction**

The objectives of instruction in the basic mathematics syllabus are for students to

• be able to use mathematics as an aid in everyday life and social activities;

• obtain positive learning experiences when working with mathematics and learn to trust their own abilities, skills and thinking; find courage to engage in experimental, exploratory and inventive learning;

• acquire such mathematical skills, knowledge and capabilities that will create a sufficient foundation for further studies;

• internalise the significance of mathematics as a tool which can be used to describe, explain and model phenomena and to draw conclusions;

- form an overview of the nature of mathematical knowledge and its logical structure;
- gain practice in receiving and analysing information provided by the media in a mathematical form and in assessing its reliability;

• acquaint themselves with the significance of mathematics in the development of culture;

• learn to use figures, formulae and models in support of thinking. (129)

## Mathematics, advanced syllabus

The role of instruction in the advanced mathematics syllabus is to provide students with the mathematical capabilities required in vocational studies and higher education. In advanced mathematics studies, students will be given opportunities to adopt mathematical concepts and methods and to learn to understand the nature of mathematical knowledge. In addition, instruction will aim to give students a clear understanding of the significance of mathematics to the development of society and of its applications in everyday life, science and technology.

**Objectives** of instruction

The objectives of instruction in the advanced mathematics syllabus are for students to

• become accustomed to persistent work, thus learning to trust their own mathematical abilities, skills and thinking;

• find courage to adopt experimental and exploratory approaches, discover solutions and assess these critically;

• understand and be able to use mathematical language, so as to be capable of following mathematical presentations, reading mathematical texts and discussing mathematics, and learn to appreciate precision of presentation and clarity of argumentation;

• learn to perceive mathematical knowledge as a logical system;

• develop their skills to process expressions, draw conclusions and solve problems;

gain practice in processing information in a way characteristic of mathematics, become accustomed to making assumptions, examining their validity, justifying their reasoning and assessing the validity of their arguments and the generalisability of the results;
gain practice in modelling practical problem situations and making use of various problem-solving strategies;

know how to use appropriate mathematical methods, technical aids and information sources.
(122-123)

#### Structure:

School structure:

When Finnish children turn 7 years old they go into compulsory primary school for nine years. All kids start at the same level, no matter what socio-economic background they have. Students then have the choice between general upper secondary or vocational upper secondary.

General upper secondary education continues the teaching and educational tasks of basic education to students aged about 16–19. The general upper secondary school ends in the matriculation examination which yields eligibility for all higher education studies.

Students in vocational upper secondary education and training are mainly aged 16-25 years. The school-based education system means full-time studies for three years at a vocational institution. Education and training of compulsory studies is primarily organized in year classes.

For upper secondary schools, upper secondary school studies are divided into compulsory, specialization and applied courses. Students in general upper secondary education intended for young people are usually 16 to 19 years of age. General upper secondary education is mainly organized without division into grades and teaching is not tied to year classes. The scope of the syllabus is three years but the studies may be accomplished in two, three or four years; the students may proceed in their studies either as a group or individually.

For the mathematics courses, there are two main groupings: basic and advanced Mathematics. Within these groupings, there are two sub-groupings: compulsory and specialized courses. The information for these courses is split into two categories: objectives and core content. Although the courses are seperated by different content, curriculum organizers within or across the courses are not used.

## Learning Outcomes (Learning Standards)

The Finnish National Board of Education decides on the objectives and core contents of the different subjects, subject groups, thematic subject modules and student counseling (national core curriculum).

For the mathematics courses, there are two main groupings: basic and advanced Mathematics. Within these groupings, there are two sub-groupings: compulsory and specialized courses. The information for these courses is split into two categories: objectives and core content. Although the courses are seperated by different content, curriculum organizers within or across the courses are not used.

The objectives are listed, in no particular order, and are not numbered. The objectives are quite broad in their content.

Example:

2. Geometry (MAB2) - compulsory course, under the basic Mathematics pathway

The objectives of the course are for students to

• gain practice in making observations and drawing conclusions about the geometrical properties of figures and bodies;

- reinforce their skills in drawing plane figures and three-dimensional bodies;
- know how to solve practical problems using geometry.

For the compulsory course, 1. Expressions and Equations (MAB1), under the basic Mathematics pathway, there are three objectives (129).

For the specialization course, 11. Number Theory and Logic (MAA11), under the advanced Mathematics pathway, there are six objectives (127-128).

#### Learning Contexts

Teaching situations in mathematics will be organized in such a way as to inspire students to ask questions, make assumptions and draw conclusions based on their observations, and to justify these. In particular, students will be guided to perceive the meanings of mathematical concepts and to recognize the ways in which these are connected to larger wholes. Students will also be encouraged to develop creative solutions to mathematical problems. Instruction will examine connections between mathematics and everyday life and make conscious use of any opportunities that may arise to develop students' personalities. This means, among other things, guiding their interest, encouraging them to experiment and developing their information acquisition processes. Resources permitting, the flexibility of the course descriptions may be used to consolidate the core contents and form integrative modules. (122)

## Competencies

#### Key competencies in basic and upper secondary education:

Embedded within the relevant disciplines:

- Communication
- Mathematical competence
- Basic competence in science
- Technology
- Social and civic competencies
- Cultural awareness and expression

Belonging to the aims of all subjects and cross-curricular themes: Digital competence, learning to learn, sense of initiative and entrepreneurship

## Key competencies in vocational education and training:

- Communication in the mother tongue
- Technology and IT
- Learning and problem-solving skills
- Interaction and cooperation skills
- A sense of initiative and entrepreneurship
- Active citizenship and different cultures

## **CROSS-CURRICULAR THEMES**

Cross-curricular themes are educational challenges with social significance. At the same time, they are current statements on values. In practical terms, cross-curricular themes are policies that structure the upper secondary school's operational culture and priority areas that cross subject boundaries and integrate education. They deal with issues concerning the way of life as a whole.

The objectives uniting all cross-curricular themes are for students to be able to:

- observe and analyse contemporary phenomena and operating environments;
- express justified ideas of a desirable future;
- assess their own lifestyle and prevailing trends from a future perspective; and
- make choices and take action for the future that they consider as being desirable.

The cross-curricular themes common to all upper secondary schools are:

- active citizenship and entrepreneurship;
- safety and well-being;
- sustainable development;
- cultural identity and knowledge of cultures;
- technology and society;
- communication and media competence.

## Strengths and Limitations

A strength in the Mathematics curriculum is the openness of the outcomes; although this is a potential difficulty for beginning teachers to manage, it seems that Finland provides a great deal of teaching experience in their teacher education programs, in addition to requiring a Master's degree.

A limitation in the curriculum is the organization of the material on the website, as it is a bit cumbersome to navigate.

# **Cross-Jurisdictional Summary**

Area of Learning: Mathematics

 $\label{eq:linear} \begin{array}{l} \mbox{Jurisdiction, year of publication and url: Australia, 2012,} \\ \mbox{http://www.australiancurriculum.edu.au/Australian%20Curriculum.pdf?a=M&l=F&l=1&l=2&l=3&l=4&l} \\ \mbox{=}5&l=6&l=7&l=8&l=9&l=10&l=10&A&e=0&e=1&e=2&e=3&e=4&e=5&e=6&e=7&kx=0 \\ \end{array}$ 

Grade level (s) reviewed: F-10

## Intentions and Rationale

Learning mathematics creates opportunities for and enriches the lives of all Australians. The Australian Curriculum: Mathematics provides students with essential mathematical skills and knowledge in Number and Algebra, Measurement and Geometry, and Statistics and Probability.

It develops the numeracy capabilities that all students need in their personal, work and civic life, and provides the fundamentals on which mathematical specialties and professional applications of mathematics are built. Mathematics has its own value and beauty and the Australian Curriculum: Mathematics aims to instill in students an appreciation of the elegance and power of mathematical reasoning. Mathematical ideas have evolved across all cultures over thousands of years, and are constantly developing. Digital technologies are facilitating this expansion of ideas and providing access to new tools for continuing mathematical exploration and invention. The curriculum focuses on developing increasingly sophisticated and refined mathematical understanding, fluency, logical reasoning, analytical thought and problem solving skills. These capabilities enable students to respond to familiar and unfamiliar situations by employing mathematical strategies to make informed decisions and solve problems efficiently. The Australian Curriculum: Mathematics ensures that the links between the various components of mathematics, as well as the relationship between mathematics and other disciplines, are made clear. Mathematics is composed of multiple but interrelated and interdependent concepts and systems which students apply beyond the mathematics classroom.

The curriculum anticipates that schools will ensure all students benefit from access to the power of mathematical reasoning and learn to apply their mathematical understanding creatively and efficiently. The mathematics curriculum provides students with carefully paced, in-depth study of critical skills and concepts. It encourages teachers to help students become self-motivated, confident learners through inquiry and active participation in challenging and engaging experiences.

Aims

The Australian Curriculum: Mathematics aims to ensure that students:

- are confident, creative users and communicators of mathematics, able to investigate, represent and interpret situations in their personal and work lives and as active citizens
- develop an increasingly sophisticated understanding of mathematical concepts and fluency with processes, and are able to pose and solve problems and reason in Number and Algebra, Measurement and Geometry, and Statistics and Probability
- recognize connections between the areas of mathematics and other disciplines and appreciate mathematics as an accessible and enjoyable discipline to study.

(3)

## Structure:

Although the curriculum is described year by year, this document provides advice across four year groupings on the nature of learners and the relevant curriculum:

- Foundation Year 2: typically students from 5 to 8 years of age
- Years 3–6: typically students from 8 to 12 years of age
- Years 7–10: typically students from 12 to 15 years of age
- Senior secondary years: typically students from 15 to 18 years of age.

## (6)

For the grade groupings of Foundation -9, there is one pathway at each level.

For Mathematics 10, there are two options: 10 and 10A.

The 10A content is optional and is intended for students who require more content to enrich their mathematical study whilst completing the common Year 10 content. It is NOT anticipated that all students will attempt the 10A content, but doing so would be advantageous for students intending to pursue Mathematical Methods (Course C) or Specialist Mathematics (Course D) in the senior secondary years. A selection of topics from the 10A curriculum can be completed according to the needs of the students.

It is anticipated that all students will study the Australian Curriculum: Mathematics up to the end of Year 10. From Year 10, the curriculum should provide pathway options suitable for students of differing abilities and interests, and with a range of future career and study plans.

Four mathematics courses have been designed for the senior secondary years. They have been designed to allow flexibility for students, taking into account a range of future pathways and the reality that some students reassess their choice of mathematics program part way through the senior secondary years. (7-8)

(\* -)

The four courses are:

- Essential Mathematics
- General Mathematics
- Mathematical Methods
- Specialist Mathematics

The Australian Curriculum: Mathematics is organized around the interaction of three content strands and four proficiency strands.

The content strands are Number and Algebra, Measurement and Geometry, and Statistics and Probability. They describe what is to be taught and learnt.

The proficiency strands are Understanding, Fluency, Problem Solving, and Reasoning. They describe how content is explored or developed, that is, the thinking and doing of mathematics. They provide the language to build in the developmental aspects of the learning of mathematics and have been incorporated into the content descriptions of the three content strands described above. This approach has been adopted to ensure students' proficiency in mathematical skills develops throughout the curriculum and becomes increasingly sophisticated over the years of schooling.

(4)

## Learning Outcomes (Learning Standards)

Across Foundation to Year 10, achievement standards indicate the quality of learning that students should typically demonstrate by a particular point in their schooling. Achievement standards comprise a written description and student work samples.

An achievement standard describes the quality of learning (the extent of knowledge, the depth of understanding, and the sophistication of skills) that would indicate the student is well placed to commence the learning required at the next level of achievement. The sequence of achievement standards across Foundation to Year 10 describes progress in the learning area. This sequence provides teachers with a framework of growth and development in the learning area. Student work samples play a key role in communicating expectations described in the achievement standards. Each work sample includes the relevant assessment task, the student's response, and annotations identifying the quality of learning evident in the student's response in relation to relevant parts of the achievement standard. Together, the description of the achievement standard and the accompanying set of annotated work samples help teachers to make judgments about whether students have achieved the standard. (8)

Under each content strand (curriculum organizer) is a sub-organizer, and then a list of learning standards. They are not numbered or listed in any particular order.

For Year One (Grade One), there are 15 learning standards.

For Year Nine, there are 25 learning standards.

#### Learning Contexts

Australian students have multiple, diverse, and changing needs that are shaped by individual learning histories and abilities as well as personal, cultural and language backgrounds and socioeconomic factors. ACARA is committed to the development of a high quality curriculum for all Australian students that promotes excellence and equity in education. Teachers will use the Australian Curriculum to develop teaching and learning programs that build on student's current learning and which are not limited by an individual student's gender, language, sexual orientation, pregnancy, culture, ethnicity, religion, health or disability, socio economic background or geographic location. The Australian Curriculum is shaped by the propositions that each student can learn and that the needs of every student are important. The flexibility offered by the Australian Curriculum enables teachers to plan rigorous, relevant and engaging learning and assessment experiences for all students. The Australian Curriculum sets out the sequence of learning typically expected across the years of schooling Foundation to Year 10. The curriculum content, presented as content descriptions, specifies the knowledge, understanding and skills that young people are to be taught and are expected to learn across the years of schooling F - 10. Teachers make flexible use of instructional processes and assessment strategies to ensure that all students are able to access, and engage with the Australian Curriculum in ways that are rigorous, relevant and meaningful. The achievement standards describe a broad sequence of expected learning in terms of what students are typically able to understand and able to do. Teachers use the achievement standards to locate the students' current levels of achievement and then plan programs that build on, and account for the different abilities of students, their prior learning experiences, cultural and linguistic backgrounds, and the different rates at which they learn.

(8-9)

## Competencies

In the Australian Curriculum, the general capabilities encompass the knowledge, skills, behaviors and dispositions that, together with curriculum content in each learning area and the cross-curriculum priorities, will assist students to live and work successfully in the twenty-first century.

There are seven general capabilities:

- Literacy
- Numeracy
- Information and communication technology (ICT) capability
- Critical and creative thinking
- Personal and social capability
- Ethical behavior
- Intercultural understanding.

In the Australian Curriculum: Mathematics, general capabilities are identified wherever they are developed or applied in content descriptions. They are also identified where they offer opportunities to add depth and richness to student learning through content elaborations. Icons indicate where general capabilities have been identified in Mathematics content. Teachers may find further opportunities to incorporate explicit teaching of the capabilities depending on their choice of activities.

(10)

There are three cross curriculum priorities in the Australian Curriculum:

- Aboriginal and Torres Strait Islander histories and cultures
- Asia and Australia's engagement with Asia
- Sustainability.

The cross curriculum priorities are embedded in the curriculum and will have a strong but varying presence depending on their relevance to each of the learning areas (12).

## Strengths and Limitations

The website is easy to navigate, and it allows the user to print specific sections for different grade levels.

A limitation is the lack of information available for the upper secondary courses, as they are still in a draft and consultation phase.

# **Cross-Jurisdictional Summary**

Area of Learning: Mathematics

Jurisdiction, year of publication and url:

Grade level (s) reviewed:

## Intentions and Rationale

(Include aims, goals, or whatever terms used to describe the overall intentions of the learning area).

(Philosophical underpinnings/ research base?)

(Briefly summarize the stated or implied rationale – why is the learning area important?)

#### Structure:

(Grade/age/year/level clusters (e.g., grades K-5, etc) Is there a rationale for this?}

(Curriculum Organizers- Identify and briefly describe the organizers that frame the curriculum and head the learning outcomes).

## Learning Outcomes (Learning Standards)

(Briefly describe the how learning outcomes or standards are structured [organizers and suborganizers, etc]. Describe the complexity of the wording. Include a count of the number of outcomes in 2 grades).

Are learning outcomes prescribed, authorized or suggested? Are there higher and lower level outcomes/standards/statements?

## Learning Contexts

(If included, identify preferred contexts such as interdisciplinary, personalized, out-of-doors).

# **Competencies**

(What are they? How are they defined? How are they integrated? How do the competencies fit within the framework? Are they cross-curricular, subject-specific or both?).

Strengths and Limitations

(Identify one strength and one limitation of the curriculum; identify any unique features

# **Cross-Jurisdictional Summary**

Area of Learning: Mathematics

Jurisdiction, year of publication and url: BC, 2007, http://www.bced.gov.bc.ca/irp/pdfs/mathematics/2007mathk7.pdf

Grade level (s) reviewed: K-7

#### Intentions and Rationale

The aim of Mathematics K to 7 is to provide students with the opportunity to further their knowledge, skills, and attitudes related to mathematics. A key component in successfully developing numeracy is making connections to these backgrounds and experiences (11).

#### Structure:

	The Mathematics curriculum is split into: K-7, 8-9, and then three different levels for 10-12. Mathematics K to 7 represents the first formal steps that students make towards becoming life-
	long learners of mathematics (14). A curriculum organizer consists of a set of prescribed learning
	outcomes that share a common focus. The prescribed learning outcomes for Mathematics K to 7
	progress in age-appropriate ways, and are grouped under the following curriculum organizers and
	suborganizers:
	Number
	Patterns and Relations
	-Patterns
	-Variables and Equations
	Shape and Space
	-Measurement
	-3-D Objects and 2-D Shapes
	-Transformations
	Statistics and Probability
	-Data Analysis
	-Chance and Uncertainty
	(15)
	Considerations for Program Delivery are also included, which contains information about:
	alternative delivery policy
	<ul> <li>inclusion, equity, and accessibility for all learners</li> </ul>
	<ul> <li>working with the Aboriginal community</li> </ul>
	<ul> <li>information and communications technology</li> </ul>
	• copyright and responsibility
	<ul> <li>fostering the development of positive attitudes</li> </ul>
	instructional focus
	• applying mathematics
	(29-34)
12	

## Learning Outcomes (Learning Standards)

For K-7, the structure is as follows: curricular organizers – suborganizers – key concepts – prescribed learning outcomes (PLO's). Schools have the responsibility to ensure that all prescribed learning outcomes in this curriculum are met; however, schools have flexibility in determining how delivery of the curriculum can best take place. It is expected that student achievement will vary in relation to the learning outcomes. Evaluation, reporting, and student placement with respect to these outcomes are dependent on the professional judgment and experience of teachers, guided by provincial policy. All learning outcomes are prescribed, and complete the stem, "It is expected that students will ...." The lists of learning outcomes are not exhaustive, and teachers may add learning that falls under the general requirement set out by the outcome (37).

The wording of the outcomes is not complex, but could not be easily understood by the students at the corresponding level. PLO's are presented by both grade level and by curriculum organizer.

Grade one: 18 PLO's; Transformations, Statistics and Probability; and Chance and Uncertainty are stated to be not applicable at this grade level (42). Grade six: 26 PLO's (56-58)

## Learning Contexts

- Students learn by attaching meaning to what they do and need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract.
- The use of a variety of manipulatives and pedagogical approaches to address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with a variety of materials, tools and contexts when constructing meaning about new mathematical ideas.
- Meaningful student discussions can provide essential links among concrete, pictorial and symbolic representations of mathematics.
- As facilitators of learning educators are encouraged to highlight mathematics concepts as they occur within the K to 7 school environment and within home environments.
- Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must realize that it is acceptable to solve problems in different ways and that solutions may vary.
- Positive learning experiences build self-confidence and develop attitudes that value learning mathematics. Working within the affective domain; recognizing the relationship between it and cognitive development.

• Importance of respecting and understanding the learning needs of the Aboriginal student

(11-12)

## Competencies

The common curriculum framework incorporates the following interrelated mathematical processes: communications; connections; mental mathematics and estimation; problem solving; reasoning; technology; and visualization. It is intended that they permeate the teaching and learning of mathematics.

- Students are expected to:
- use communication in order to learn and express their understanding

• make *connections* among mathematical ideas, other concepts in mathematics, everyday experiences and other disciplines

- demonstrate fluency with mental mathematics and estimation
- develop and apply new mathematical knowledge through problem solving
- develop mathematical reasoning
- select and use *technology* as a tool for learning and solving problems

• develop *visualization* skills to assist in processing information, making connections and solving problems.

#### Strengths and Limitations

A notable strength is the inclusion of a Student Achievement section, which provides information about classroom assessment and student achievement, including specific achievement indicators that may be used to assess student performance in relation to each prescribed learning outcome. Also included in this section are key elements – descriptions of content that help determine the intended depth and breadth of prescribed learning outcomes (81).

A limitation would be that many of the PLO's involve multiple processes, and could be split into separate functions of learning.

For example, within the grade one PLO's: B1 demonstrate an understanding of repeating patterns (two to four elements) by -describing -reproducing -extending -creating patterns using manipulatives, diagrams, sounds, and actions [C, PS, R, V]

This extends the actual number of prescribed learning outcomes that a teacher would cover (42).

# **Cross-Jurisdictional Summary**

Area of Learning: Mathematics

Jurisdiction, year of publication and url: BC, 2008, http://www.bced.gov.bc.ca/irp/pdfs/mathematics/WNCPmath1012/2008math1012wncp\_ccf.pdf

Grade level (s) reviewed: 10-12

## Intentions and Rationale

The main goals of mathematics education are to prepare students to:

- solve problems
- communicate and reason mathematically
- make connections between mathematics and its applications
- become mathematically literate
- appreciate and value mathematics
- make informed decisions as contributors to society.

Students who have met these goals:

- gain an understanding and appreciation of the
- role of mathematics in society
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical problem solving
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity about mathematics and situations involving mathematics.

In order to assist students in attaining these goals, teachers are encouraged to develop a classroom atmosphere that fosters conceptual understanding through:

- taking risks
- thinking and reflecting independently
- sharing and communicating mathematical understanding
- solving problems in individual and group projects
- pursuing greater understanding of mathematics
- appreciating the value of mathematics throughout history.
- (4)

## Structure:

The main structure of BC's mathematics 10-12 curriculum comes directly from the Western and Northern Canadian Protocol (WNCP) Common Curriculum Framework for Grades 10-12 Mathematics.

For the WNCP's grades 10-12 mathematics, the topics are split into three different pathways: Apprentice and Workplace; Foundations of Mathematics; and Pre-Calculus. Each of these pathways have their own curricular organizers, general outcomes, specific outcomes, and achievement indicators. These are guided by: 1) Nature of Mathematics: change, constancy, number sense, patterns, relationships, spatial sense, uncertainty; and, 2) Mathematical Processes: communication; connections, mental mathematics and estimation; problem solving; reasoning; technology; and visualization (5).

Note: For the 10 level, there are only two options: Apprentice and Workplace 10 or Foundations of Mathematics and Pre-Calculus 10. The Foundations of Mathematics and Pre-Calculus courses then split at the 11 and 12 levels.

The curriculum organizers for Apprenticeship and Workplace are:

10: measurement, geometry, number, and algebra

11: measurement, geometry, number, algebra, and statistics

12: measurement, geometry, number, algebra, statistics, and probability (19-39)

The curriculum organizers for Foundations of Mathematics and Pre-Calculus 10: measurement; algebra and number; and relations and functions

The curriculum organizers for Foundations of Mathematics are:

11: measurement; geometry; logical reasoning; statistics; relations and functions; and mathematics research project

12: financial mathematics; logical reasoning; probability; relations and functions; mathematics research project (47-67)

The curriculum organizers for Pre-Calculus are:

11: algebra and number; trigonometry; and relations and functions

12: trigonometry; relations and functions; and permutations, combinations, and binomial theorem

(77-91)

An additional course is Calculus 12, for which there is an Integrated Resource Package that was extracted in 2012 from the 2000 Mathematics 10-12 curriculum.

Calculus 12 is intended for students who have completed (or are concurrently taking) Principles of Mathematics 12 or who have completed an equivalent college preparatory course that includes algebra, geometry, and trigonometry. The curriculum organizers for Calculus 12 are: problem solving; overview and history of calculus; functions, graphs, and limits (functions and their graphs)(limits); the derivative (concept and interpretations); the derivative (computing derivatives); applications of derivatives (derivatives and the graph of the function); applications of derivatives (applied problems); antidifferentiation (recovering functions from their derivatives); and antidifferentiation (191).

## Learning Outcomes (Learning Standards)

For the WNCP's grades 10-12 mathematics, the topics are split into three different pathways: Apprentice and Workplace; Foundations of Mathematics; and Pre-Calculus. Each of these pathways have their own curricular organizers, general outcomes, specific outcomes, and achievement indicators.

Each curricular organizer has one corresponding general outcome. This then has a number of corresponding specific outcomes, numbered using whole numbers: 1, 2, 3, etc. Each specific outcomes has a number of related achievement indicators, using the whole number from the specific outcome and then numbered in the tenth decimal place, such as: 1.1, 1.2, 1.3, etc.

Apprenticeship and Workplace 10: 13 specific outcomes (19-27) Foundations of Mathematics and Pre-Calculus 10: 18 specific outcomes (47-55)

The wording is not complex. The specific outcomes are prescribed, but the achievement indicators are suggested.

The general outcome expands meaning of the curriculum organizer; for example, Pre-Calculus 12 Curriculum organizer: Algebra and Number General outcome: Develop algebraic reasoning and number sense (77)

## Learning Contexts

Students are curious, active learners with individual interests, abilities, needs and career goals. They come to school with varying knowledge, life experiences, expectations and backgrounds. A key component in developing mathematical literacy in students is making connections to these backgrounds, experiences, goals and aspirations. Students construct their understanding of mathematics by developing meaning based on a variety of learning experiences. This meaning is best developed when learners encounter mathematical experiences that proceed from simple to complex and from the concrete to the abstract. The use of manipulatives, visuals and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students. At all levels of understanding, students benefit from working with a variety of materials, tools and contexts when constructing meaning about new mathematical ideas. Meaningful student discussions also provide essential links among concrete, pictorial and symbolic representations of mathematics. The learning environment should value, respect and address all students' experiences and ways of thinking, so that students are comfortable taking intellectual risks, asking questions and posing conjectures. Students need to explore mathematics through solving problems in order to continue developing personal strategies and mathematical literacy. It is important to realize that it is acceptable to solve problems in different ways and that solutions may vary depending upon how the problem is understood.

(2)

Each pathway in The Common Curriculum Framework for Grades 10–12 Mathematics is arranged by topics. Students should be engaged in making connections among concepts both within and across topics to make mathematical learning experiences meaningful. Teachers should consider the following points when planning for instruction and assessment:

• The mathematical processes that are identified with the outcome are intended to help teachers select effective pedagogical approaches for the teaching and learning of the outcome.

• All seven mathematical processes must be integrated throughout teaching and learning approaches, and should support the intent of the outcomes.

• Wherever possible, meaningful contexts should be used in examples, problems and projects.

• Instruction should flow from simple to complex and from concrete to abstract.

• The assessment plan for the course should be a balance of assessment for learning, assessment as learning and assessment of learning.

The focus of student learning should be on developing a conceptual and procedural understanding of mathematics. Students' conceptual understanding and procedural understanding must be directly related.

(17)

For Calculus 12: when delivering the prescribed curriculum, teachers may freely adjust the instructional time to meet their students' diverse needs. These estimated instructional times have been recommended by the IRP writers to assist their colleagues; they are suggestions only (191).

## Competencies

The common curriculum framework incorporates the following interrelated mathematical processes: communications; connections; mental mathematics and estimation; problem solving; reasoning; technology; and visualization. It is intended that they permeate the teaching and learning of mathematics.

Students are expected to:

- use *communication* in order to learn and express their understanding
- make *connections* among mathematical ideas, other concepts in mathematics, everyday experiences and other disciplines
- demonstrate fluency with mental mathematics and estimation
- develop and apply new mathematical knowledge through *problem solving*
- develop mathematical reasoning
- select and use *technology* as a tool for learning and solving problems

• develop *visualization* skills to assist in processing information, making connections and solving problems.

(6)

## Strengths and Limitations

Following the WNCP does allow for some consistency amongst the Western provinces; however, this may not be the most educationally-sound argument for the decision. Furthermore, provinces like Alberta and Saskatchewan have derived their mathematics curriculum from the WNCP, but they have altered some of the outcomes.

Some of the other potential limitations include:

-different curriculum organizers for different grades within the same pathway -by splitting up the curriculums into three different groupings that are based on two different creation methods (BC based and WNCP), BC may be losing some consistency and continuity amongst the K-12 curriculum.

-Calculus 12 as a separate course; it is not part of the WNCP 10-12 courses.

# **Cross-Jurisdictional Summary**

Area of Learning: Mathematics

Jurisdiction, year of publication and url: BC, 2008, http://www.bced.gov.bc.ca/irp/pdfs/mathematics/2008math89.pdf

Grade level (s) reviewed: 8 and 9

## Intentions and Rationale

The aim of Mathematics 8 and 9 is to provide students with the opportunity to further their knowledge, skills, and attitudes related to mathematics. A key component in successfully developing numeracy is making connections to these backgrounds and experiences (11).

#### Structure:

The Mathematics curriculum is split into: K-7, 8-9, and then three different levels for 10-12. Mathematics 8 and 9 continues the path students begin in Mathematics K to 7 toward becoming lifelong learners of mathematics. A curriculum organizer consists of a set of prescribed learning outcomes that share a common focus. The Prescribed Learning Outcomes for Mathematics 8 and 9 progress in age-appropriate ways, and are grouped under the following curriculum organizers and suborganizers:

Number

Patterns and Relations

- Patterns

- Variables and Equations

Shape and Space

- Measurement
- 3-D Objects and 2-D Shapes
- Transformations
- Statistics and Probability

- Data Analysis

- Chance and Uncertainty

(15)

Considerations for Program Delivery are also included, which contains information about:

- alternative delivery policy
- inclusion, equity, and accessibility for all learners
- working with the Aboriginal community
- information and communications technology
- copyright and responsibility
- fostering the development of positive attitudes
- instructional focus
- applying mathematics

(29-35)

## Learning Outcomes (Learning Standards)

For 8 and 9, the structure is as follows: curricular organizers – suborganizers – key concepts – prescribed learning outcomes (PLO's). Schools have the responsibility to ensure that all prescribed learning outcomes in this curriculum are met; however, schools have flexibility in determining how delivery of the curriculum can best take place. It is expected that student achievement will vary in relation to the learning outcomes. Evaluation, reporting, and student placement with respect to these outcomes are dependent on the professional judgment and experience of teachers, guided by provincial policy. All learning outcomes are prescribed, and complete the stem, "It is expected that students will …." The lists of learning outcomes are not exhaustive, and teachers may add learning that falls under the general requirement set out by the outcome (39).

The wording of the outcomes is not complex, but could not be easily understood by the students at the corresponding level. PLO's are presented by both grade level and by curriculum organizer.

Grade eight: 17 PLO's (42-43) Grade nine: 22 PLO's (44-46)

#### Learning Contexts

- Students learn by attaching meaning to what they do and need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract.
- The use of a variety of manipulatives and pedagogical approaches to address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with a variety of materials, tools and contexts when constructing meaning about new mathematical ideas.
- Meaningful student discussions can provide essential links among concrete, pictorial and symbolic representations of mathematics.
- As facilitators of learning educators are encouraged to highlight mathematics concepts as they occur within the 8 and 9 school environment and within home environments.
- Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must realize that it is acceptable to solve problems in different ways and that solutions may vary.
- Positive learning experiences build self-confidence and develop attitudes that value learning mathematics. Working within the affective domain; recognizing the relationship between it and cognitive development.

• Importance of respecting and understanding the learning needs of the Aboriginal student (11-12)

## **Competencies**

The common curriculum framework incorporates the following interrelated mathematical processes: communications; connections; mental mathematics and estimation; problem solving; reasoning; technology; and visualization. It is intended that they permeate the teaching and learning of mathematics. Students are expected to:

• use *communication* in order to learn and express their understanding

• make *connections* among mathematical ideas, other concepts in mathematics, everyday experiences and other disciplines

- demonstrate fluency with mental mathematics and estimation
- develop and apply new mathematical knowledge through problem solving
- develop mathematical reasoning
- select and use *technology* as a tool for learning and solving problems

• develop *visualization* skills to assist in processing information, making connections and solving problems.

## Strengths and Limitations

Splitting the curriculum to focus on only two grade levels, 8 and 9, is both a strength and a limitation. It is a strength in that it allows the focus to remain on these levels only, but, as a limitation, it ignores the context of the formative and consequent learning. Thus, it does not allow for an easier 'big picture' view of the Mathematics curriculum as a whole.

# **COMPETENCIES IN MATHEMATICE - AUSTRALIA**

#### Communication http://vels.vcaa.vic.edu.au/maths/relationships.html#show hide3

Mathematical structure and *Working mathematically* play essential roles in understanding natural and human worlds. In communicating about these worlds, students use a combination of everyday language and mathematical symbols involving numerals, operations, connectives, variables and relations.

Development of the language of Mathematics is crucial to its practical application. Students learn to use the language and concepts of Mathematics both within the discipline itself, and also its applications to modelling and problem solving across the other domains. In this process they employ a range of Communication tools for illustrating relationships and displaying results such as venn diagrams and tree diagrams.

#### English

The use of logical and analytical thinking in Mathematics, including the use of conjectures and proof, has clear links to the development of structured and coherent argument in speaking and writing. Mathematical structure is strongly related to semantics, syntax and language, and to the use of propositions and quantifiers embedded in principled argument in natural language.

The development of skills for critical analysis of literary, everyday and media texts in English empowers individuals to participate effectively in society. This is complemented by the fundamental role that Mathematics plays in cultural, social and technological advances and in empowering individuals as critical citizens in contemporary society and for the future.

Number, space and measurement, chance and data, are common aspects of people's experience in everyday personal, study and work situations, and are naturally embedded in activities related to the English dimensions of reading, writing, speaking and listening.

#### Language Other Than English (LOTE)

Common and central to both domains is consideration in language of semantic (meaning of key concepts, ideas, terms) and syntactic (structural relationships in natural and symbolic language) elements and features.

This involves acquiring knowledge of words, symbols, terms and definitions, understanding and application of concepts, and the use of related technical skills. Students develop thinking processes of analysis and synthesis, involving relationships between simple and complex ideas and expressions.

These developments involve the use of Mathematics in everyday activities such as number systems in counting, buying and selling, measuring, designing and building, and estimating and describing chance events using different written and spoken languages in a range of social and cultural contexts.

#### Thinking Processes http://vels.vcaa.vic.edu.au/maths/relationships.html#show\_hide3

The study of Mathematics exemplifies important thinking processes within its own discipline and also provides a context within which these and other thinking processes can be developed and refined. Mathematical reasoning and thinking underpins all aspects of school mathematics, including problem posing, problem solving, investigation
and modelling. It encompasses the development of algorithms for computation, formulation of problems, making and testing conjectures and the development of abstractions for further investigation. Computation and proof are essential and complementary aspects of Mathematics that enable students to develop thinking skills directed toward explaining, understanding and using mathematical concepts, structures and objects.

Thinking strategies and tools are used extensively in Mathematics. As students progress in their learning they move from using concrete thinking skills to applying higher order processes to their learning. Students are encouraged to take managed risks in developing possible alternate approaches to problems and tasks. Curiosity has an important role to play in stimulating mathematical inquiry, while reflection and metacognition are important components of the problem solving process in which new and often creative approaches are required to find solutions.

#### Design, Creativity and Technology http://vels.vcaa.vic.edu.au/maths/relationships.html#show\_hide3

In Design, Creativity and Technology students apply numbers to everyday situations; they explain and use mental and written algorithms for addition, subtraction, multiplication and division. Students carry out arithmetic computations and choose mathematical models and procedures to apply in the process of designing and making products and systems.

When designing, students use drawing tools, accurately drawing two-dimensional and three-dimensional representations. They represent depth in their drawings and describe what can and cannot be seen in simple objects and drawings. They use a variety of forms of representation including oblique, isometric and perspective to visually represent objects and use computer drawing packages and computer aided design/computer-aided manufacturing (CAD/CAM) for two- and three-dimensional representations and analysis of design.

In the process of investigating, designing, making and testing products and systems students measure length, perimeter, area, surface area, angle and time for familiar products. They recognise and use different units of measurement for appropriate contexts and use suitable instruments to measure length, angle, volume and time. They select and use suitable procedures to measure, estimate and calculate for instance length, area, volume and angle. In the process they learn to use formulae as appropriate. Students investigate situations and solve problems set in a wide range of practical contexts, for example, the design and construction for packaging involving the use of nets for three dimensional shapes.

#### Interpersonal Development http://vels.vcaa.vic.edu.au/maths/relationships.html#show\_hide3

 In working mathematically students will engage with others to formulate and test conjectures, gather, analyse and interpret data and apply mathematics to solve real life problems. These activities are often carried out in teams that require students to collaborate and cooperate, share and discuss, and these behaviours are central in working towards the standards in Interpersonal Development.

Personal Learning http://vels.vcaa.vic.edu.au/maths/relationships.html#show\_hide3

The mathematical processes of inquiry, investigation, problem-solving, modelling and the use of technology give rise to opportunities and challenges for Personal Learning. These mathematical processes provide contexts within which students can acquire self knowledge and dispositions that support learning. Opportunities also occur for students to learn with their peers including seeking and responding appropriately to feedback. Establishing values in which individual differences are respected and appreciated encourages unique and varied approaches to working mathematically. As students see mathematical connections and are able to apply mathematical concepts, skills and processes in posing and solving problems, they become confident in their personal knowledge of Mathematics. They increasingly manage their own learning and growth through the setting of goals and managing resources. The strategies of planning, monitoring, revising, reflecting and if needs be modifying enable students to develop resilience and become adaptive learners. Students will become progressively empowered through knowledge of mathematics and as numerate citizens they will be able to apply this knowledge critically, in societal and political contexts.

#### SOCIAL RESPONSIBILITY (from Concept Meeting August 16<sup>th</sup> & 17<sup>th</sup>, 2012)

- Critical mathematics; It takes math to comprehend the magnitude of problems.
  - Informing about global crises (links to curriculum)
    - Pollution
    - Poverty
    - recycling
- Actions that can be taken and consequences of those
- Application of appropriate mathematics in your world

#### Civics and Citizenship http://vels.vcaa.vic.edu.au/maths/relationships.html#show\_hide3

The concepts developed in the study of Mathematics are applicable to a range of Civic and Citizenship understandings. Mathematical structure and working play essential roles in key aspects of our society as well as key civics concepts. Particular aspects of Civics and Citizenship require mathematical understanding, including concepts of majority rule, absolute majority, one vote one value, representation based on electorates of equal sizes, and the preferential and proportional voting systems. Mathematical understanding also supports the development of community research and the presentation of findings, for example, the use of data and statistics, and including the analysis and presentation of information in charts and diagrams. The recognition and understanding of this mathematical basis to many social structures and processes is integral to the development of informed and active citizens.

From:	Walt, Nancy J EDUC:EX
To:	Collins, Valerie A EDUC:EX; Horner, Melissa EDUC:EX
Subject:	Council of Ministers of Education, Canada   Ministers of Education Highlight 21st-Century Literacies on International Literacy Day
Date:	Wednesday, September 11, 2013 6:19:56 AM

Expanding definition of literacies. We are using these ideas in some of the story text:

http://www.newswire.ca/en/story/1221027/ministers-of-education-highlight-21st-century-literacies-oninternational-literacy-day

# Table 4.12 Conceptions of curriculum integration in Québec

working within th	e MEQ shown in Italics.)	
Estates	Task Force's ideals for	Québec Education Program, in
General intent	achieving these intents were	response to these intents and ideals
was to: that curriculum would:		developed practices that included:
Integrate	Integrate changing societal	The broad areas (issues) of learning:
societal	issues of relevance to young	Health & well-being; Personal & career
changes – <i>an</i>	people and develop	planning; Environmental awareness &
ongoing.	conscious and critical	consumer rights & responsibilities:
organic process	reflective practices.	Media literacy: Citizenship & community
<b>J</b>	· · · · · · · · · · · · · · · · · · ·	life: (open to include "real" issues)
Integrate	Group the programs and	Subject areas (Languages: Social
disciplines into	reorganize content based on	Sciences: Arts: Math Science &
major areas of	purpose and contribution to	Technology: Personal Development)
learning <i>_fields</i>	learning rather than value or	use a common vocabulary to talk about
of knowledge	ranking of subjects	learning and the role of each subject
Integrate	Ensure generic skills are	Develop cross-curricular competencies
fundamental	mastered through cross-	(intellectual: methodological: personal
learninge	curricular learnings offer	and social: communications related) by
lifolong loorning	boginning with the	using metacognition generic models
	dissiplines with the issues or	representational evoteme process
37/113	with the competency	skille and creativity to connect locrains
liste suete	with the competency.	Skills, and creativity to connect learning.
Integrate	improve cultural content by	Constructivist model of teaching and
deeper	Integrating cultural elements	learning - openness to multiple realities,
(situated)	I nere is better retention of	resources, information sources, and
knowledge	learning when knowledge is	reflective practices to make learning
	actively constructed.	connected and meaningful.
Integrate	Accept common values and	Citizenship education and community
students into	common goals for a	life as one of the broad areas of
society as	democratic society - a social	learning. Encourage student-initiated
responsible	project recognizing diversity	projects and the collaborative processes
citizens	of contributions	of negotiation and decision-making.
Integrate	Assimilate the cultural	Media literacy and a respect for
cultural heritage	traditions of Québec's history	individual and collective rights.
	and language while retaining	Encourage community involvement and
	personal identity and a sense	community-initiated projects to promote
	of community.	visions of self and community.
		Value multiple languages.
Integrate	Fight against social exclusion	Encourage local control over how to
personal	by reducing academic failure	implement the QEP.
experiences	and adapting courses to	Encourage student input and active
•	students' interests and ways	learning experiences.
	of learning.	Examine evaluation practices.
Integrate skills	Include cohesive measures	(Not as evident in the elementary
for employment	to help each student clarify	program) Use formative evaluations and
and community	his or her academic and	emphasize learning processes in
livina	career goals.	applicable contexts.
Systemic and	Challenge the dominant	The OFP model attempts to present
organizational	model for organizing school	learning objectives and content that are
integration and	time by encouraging a cycle	linked to societal issues subject and
cohosion	has a cranization	apperie competencies. School teams
001031011	มลรธน บายลากิรสแบก.	are encouraged to adapt this document
		to suit their collective projects.

(NB: Documented conceptions are listed first with additional conceptions from educational leaders working within the MEQ shown in Italics.)

Pages 77 through 96 redacted for the following reasons: S3

# **Cross-Curricular Competencies in Mathematics**

Our 21<sup>st</sup> century learners face an uncertain road beyond their K-12 school experience. They must face this future armed with more than facts, but, instead, with mathematically nuanced ways of thinking, communicating, and acting that enable them to successfully navigate their world.

All students can learn mathematics. Not all students may learn the same mathematical content, depending on their interests, needs, and goals for transition into adulthood. Ended comparing w/ Bill's version here Engaging in mathematics enables more than the ability to calculate numbers or follow algorithms; rather, mathematics provide an opportunity for learners to observe, understand, and embody mathematical habits of mind (Appendix A) and mathematical skills and processes (Appendix B).

# **Thinking Competency**

Thinking competency in mathematics represents the knowledge, skills and processes we associate with the learner's intellectual development. It is through their competency as thinkers that students take the mathematical content and transform it into new understandings. The thinking competency includes specific mathematical thinking processes and skills; mathematical and general habits of mind; metacognitive awareness; and executive function. Together, these components of thinking competency represent the abilities students need to undertake deep and lifelong learning.

# **Critical Thinking**

Critical thinking in mathematics occurs when a learner faces a task or problem for which different strategies are available, or which has a number of possible solutions, or both. Critical thinking is not required when a learner is determining a correct answer to a routine task or problem.

The learner selects purposeful prior knowledge; creates or recalls relevant strategies; connects big and small mathematical ideas; and evaluates solutions in order to determine the best answer.

Students will be able to demonstrate their *critical thinking in mathematics* by:

- identifying prior knowledge to help solve problems
- determining a starting point when solving problems
- determining what is known and unknown
- understanding how ideas in mathematics are connected
- determining relevant information
- utilizing estimation and mental mathematics

- examining small and large cases to find and test conjectures
- deciding on a strategy for a given situation
- varying parameters in regular and useful ways
- analyzing, interpreting, and organizing data
- making predictions
- discovering patterns
- discerning between different representations
- deriving meaning from patterns
- reasoning deductively and inductively
- applying skepticism to results
- selecting a best solution
- presenting process, solutions, and final answer in a meaningful way
- justifying solutions and choice of final answer.

## **Creative Thinking**

To approach mathematics in different ways to solve problems and demonstrate understanding requires creative thinking. Visualizing, illustrating, and modeling are important ways to demonstrate conceptual understanding of mathematics. Creative mathematical thinkers take risks in order to discover possible solutions to problems and explore connections between concepts and big ideas.

Students will be able to demonstrate their creative thinking in mathematics by:

- formulating a problem to solve
- working backwards by starting with a solution
- considering different perspectives
- finding and exploiting similarities between problems
- visualizing and illustrating solutions, mathematically
- modeling problems, strategies, and solutions
- generating multiple methods to solve problems
- looking for and creating shortcuts
- generating other ways of showing the same problem
- tolerating complexity
- discovering formulas
- looking for hidden aspects
- developing algorithms
- creating mathematical conjectures
- wondering about extensions to concepts
- inquiring and identifying other applications of math concepts
- expanding on solutions
- developing abstractions and generalizations
- inventing innovative mathematical structures and systems
- presenting in non-traditional ways

- demonstrating willingness to experiment, guess, and reconsider process
- questioning and challenging their process
- discovering connections to other areas.

## **Reflective Thinking**

Reflective thinking is the ability to consider one's own thinking and learning processes. Reflection and metacognition are important components of learning, especially in the development of self-regulation. Metacognition allows students to: contemplate their understanding of mathematical concepts, evaluate the efficiency of their strategies, and consider the reasonableness and effectiveness of their solutions.

Students will be able to demonstrate their *reflective thinking in mathematics* by:

- considering prior knowledge and experiences
- continuously reflecting on process
- evaluating the reasonableness of a solution
- persevering through learning
- evaluating the efficiency/usefulness of different strategies
- comparing advantages/disadvantages of different solutions
- identifying strengths and weaknesses of their understanding
- stating assumptions
- being thoughtful and purposeful in selecting cases to explore
- analyzing their process in finding solutions
- considering the best solution within the given context
- identifying applications of math concepts
- identifying improvements for future problem solving
- evaluating and describing their thinking and learning processes.

# Personal and Social Competency

Personal and social competency in mathematics is the set of abilities that relate to students' identity as being mathematically competent and confident in the world, both as individuals and as members of their community and society.

## **Positive Personal and Cultural Identity**

A positive personal and cultural identity, in the context of mathematics, is the awareness, understanding, and appreciation of diversity, ancestry, culture, language, belief and perspective in a multi-cultural and socially diverse society. In today's classrooms, individuals bring a variety of cultural and personal perspectives and processes that enrich the mathematical experience for all. Establishing values in which individual differences are respected and appreciated encourages unique and varied approaches to working mathematically. The awareness, understanding, and appreciation of personal and cultural identity enrich the mathematical learning experience. Increased mathematical awareness helps one identify as a Mathematician and creates a belief in the ability to be a confident and capable contributor.

Students will be able to demonstrate their *positive personal and cultural identity in mathematics* by:

- appreciating mathematics in one's own life
- conveying an appreciation for the inherent beauty of mathematics
- identifying as a Mathematician
- believing in their ability to make sense of mathematics
- understanding how mathematics has contributed to the development of one's culture
- respecting personal and cultural differences, skills and knowledge, in relation to mathematics
- developing numeracy skills that nurture self-confidence
- making connections between mathematics and life.

## Personal Awareness and Responsibility

In the context of mathematics, an awareness of the numeracy skills necessary in order to function as a contributing member of society is realized. Mathematicians use mathematics to interpret problems and issues for discussions within the public domain. They make mathematics relevant to daily life; this includes self-regulation with regards to budgets and financial contexts, as well as awareness and an understanding of how we use numbers for the communication of personal health and wellness.

These mathematical processes provide contexts within which students can acquire selfknowledge and dispositions that support learning. The mathematical processes of inquiry, investigation, problem-solving, and modelling give rise to opportunities and challenges for personal learning. As students see mathematical connections and are able to apply mathematical concepts, skills and processes in posing and solving problems, they become confident in their personal knowledge of mathematics. They increasingly manage their own learning and growth through the setting of goals and managing resources, including the appropriate use of technology. The strategies of planning, monitoring, revising, reflecting and if modifying enable students to develop resilience and become adaptive learners. Students will become progressively empowered through knowledge of mathematics and as numerate citizens they will be able to apply this knowledge critically, in societal and political contexts.

Students will be able to demonstrate their *personal awareness and responsibility in mathematics* by:

- accepting that risk-taking is an important part of the learning process
- demonstrating an understanding of the value and management of money, and the consequences of financial transactions
- making decisions based on personal referents of number and measure
- explaining a variety of mathematics used in real-life applications

- making choices on the appropriate technology for solving or presenting mathematical problems
- persevering in problem solving
- asking for clarification when necessary
- recognizing that mistakes are part of the learning process
- respecting that people have different strategies for solving the same problems
- practicing good time management strategies
- choosing the appropriate tools and strategies to solve a problem
- developing an appreciation for math in artistic forms
- taking responsibility for their own mathematical learning.

# Social Awareness and Responsibility

Using the tools developed through personal awareness and responsibility, students can apply their knowledge and understanding to interpret civic, societal, and global situations mathematically. Students will be able to develop action plans and participate in discussions on issues in a problem solving manner. Application of appropriate mathematics in the world helps determine actions that can be taken and the consequences of a plan.

In working mathematically, students will engage with others to formulate and test conjectures, gather, analyse and interpret data and apply mathematics to solve real life problems. These activities are often carried out in teams that require students to collaborate and cooperate; then share and discuss. Opportunities for students to learn with their peers include asking and responding appropriately to feedback, and these behaviours are central in working towards interpersonal development.

Students will be able to demonstrate their *social awareness and responsibility in mathematics* by:

- respecting the opinions of others
- challenging others in a respectful way when there is disagreement
- ensuring others have access to participating
- demonstrating willingness to help others
- discussing and sharing ideas
- sharing work in an equitable way
- giving opportunities for others to have a moment of realization
- using mathematics to foster social awareness and responsibility with regards to global economic situations and issues
- taking action cooperatively to solve problems
- developing a sense of responsibility for their actions and the effects in the world
- using math as a tool in sustainability and social justice contexts
- describing and informing decisions for positive solutions to today's social, environmental, economic or other contextual issues
- enabling ethical use of data collection, analysis, and representation.

# **Communication Competency**

Communication competency in mathematics encompasses the set of abilities that learners use to impart and exchange information, experiences, and ideas, to explore the world around them, and to understand and effectively engage in the use of digital media. The communication competency provides a bridge between students' learning, their personal and social identity and relationships, and the world in which they interact.

# **Using Language and Symbols**

Students communicate their sense-making and reasoning of mathematical concepts not only through mathematical symbols, but also through words, pictures, models, concrete representations, actions, and combinations of. Just as students think and reason differently, they also communicate their thinking differently. It is essential that students value the importance of communicating their mathematical understanding, not just for their own benefit but also for the benefit of their classmates and society in general.

Using effective communication serves to clarify, reinforce, and modify ideas, attitudes, and beliefs in multiple global settings. Students need to be encouraged to use a variety of forms of communication while learning mathematics and to communicate their relevant connections by using mathematical terminology.

Students will be able to demonstrate their use of language and symbols in mathematics by:

- recognizing, differentiating, and effectively using mathematical symbols and languages
- using mathematical symbols and language
- explaining the mathematical processes they use
- articulating generality with precise language
- constructing viable and reasonable arguments
- justifying their reasoning mathematically
- justifying their choice of strategy to solve a problem
- describing their sense-making through concrete, pictorial, and symbolical representations
- articulating the connections between:
  - multiple representations of a concept
  - two or more concepts
  - concept(s) and the real world
- recognizing relationships visually, numerically, and algebraically
- explaining abstract and concrete events mathematically
- viewing, interpreting, and explaining mathematics in real world settings

• modeling the action of a mathematical concept(s).

# **Digital Literacy**

Students purposefully and personally engage in a vast array of digital tools that enhance their learning and doing of mathematics, as well as their ability to communicate these. Much more than being critical consumers of digital media, students are creative and share their thinking to a broader community in a personalized way. They have awareness and are open to making use of the many sources, tools and platforms available.

Students will be able to demonstrate their *digital literacy in mathematics* by:

- using appropriate digital forms when communicating their mathematical thinking
- creating, constructing, and sharing mathematical models or representations
- collaborating with others
- using digital tools within the classroom
- using virtual tools beyond the classroom
- choosing appropriate technologies
- locating digital sources and evaluating what is meaningful and useful
- investigating mathematics dynamically
- organizing and managing information
- analyzing and synthesizing.

# Appendix A: Mathematical habits of mind

## Characteristics of general habits of minds (Cuoco, Goldenberg, & Mark, 1996)

- Pattern sniffers
- Experimenters
- Describers
- Thinkers
- Inventors
- Visualizers
- Conjecturers
- Guessers

## Habits of mind specific to mathematics (Cuoco, Goldenberg, & Mark, 1996)

- Talk big and think small
- Talk small and think big
- Use functions
- Use multiple points of view
- Mix deduction and experiment
- Push the language
- Use intellectual chants
- Algebraic approaches to things
  - a) like a good calculation
  - b) use abstraction
  - c) use algorithms
  - d) break things into parts
  - e) extend things
  - f) represent things
- Geometric approaches to things
  - a) use proportional reasoning
  - b) use several languages at once
  - c) use one language for everything
  - d) love systems
  - e) worry about things that change
  - f) worry about things that do not change
  - g) love shapes

# Mathematical Habits of Mind for Young Children (Goldenberg, E. P., Shteingold, N., & Feurzeig, 2003)

- Thinking about word meaning
- Justifying claims and proving conjectures
- Distinguishing between agreement and logical necessity
- Analyzing answers, problems, and methods
- Seeking and using heuristics to solve problems

## Mathematical Habits of Mind for Secondary Students (Levasseur & Cuoco, 2003)

- Guessing
- Challenging solutions, even correct ones
- Looking for patterns
- Conserving memory
- Seeking special cases
- Using alternative representations
- Classifying carefully
- Thinking algebraically

## General Habits of Mind for High School Mathematics (Cuoco, Goldenberg, & Mark, 2010)

- Performing thought experiments
- Finding, articulating, and explaining patterns
- Creating and using representations
- Generalizing from examples
- Articulating generality in precise language
- Extracting mathematics to make sense

## Habits and Values of Mathematicians (Seaman & Szydlik, 2007, p. 170-171)

- Seek to understand patterns based on underlying structure
- Make analogies by finding the same essential structure in seemingly different mathematical objects
- Make and test conjectures about mathematical objects and structures
- Create mental (and physical) models for examples (and non-examples) of math objects
- Value precise mathematical definitions of objects
- Value an understanding of why relationships make sense
- Value logical arguments and counterexamples as our sources of conviction
- Value precise language and have fine distinctions about language
- Value symbolic representations of, and notation for, objects and ideas

# Four Groups of Reasoning Habits in NCTM's *Focus in High School Mathematics: Reasoning and Sense Making* (NCTM, 2009, p. 9-10)

1. Analyzing a problem

- Identifying relevant mathematical concepts, procedures, or representations that reveal important information about the problem and contribute to its solution
- Defining relevant variables and conditions carefully, including units if appropriate;
- Seeking patterns and relationships
- Looking for hidden structure
- Considering special cases or simpler analogs
- Applying previously learned concepts to new problem situations, adapting and extending as necessary
- Making preliminary deductions and conjectures, including predicting what a solution to a problem might involve or putting constraints on solutions
- Deciding whether a statistical approach is appropriate

## 2. Implementing a strategy

- Making purposeful use of procedures
- Organizing the solution
- Making logical deductions
- Monitoring progress toward a solution

3. Seeking and using connections across different mathematical domains, different contexts, and different representations

4. Reflecting on a solution to a problem

- Interpreting a solution and how it answer the problem
- Considering the reasonableness of a solution
- Revisiting initial assumptions about the nature of the solution, including being mindful of special cases and extraneous solutions;
- Justifying or validating a solution, including through proof or inferential reasoning;
- Recognizing the scope of inference for a statistical solution
- Reconciling different approaches to solving the problem
- Refining arguments so that they can be effectively communicated
- Generalizing a solution to a broader class of problems and looking for connections with other problems

# Standards for Mathematical Practice in *Common Core State Standards in Mathematics* (CCSSI, 2010, p. 6-8)

- Make sense of problems and persevere in solving them
- Reason abstractly and quantitatively
- Construct viable arguments and critique the reasoning of others
- Model with mathematics
- Use appropriate tools strategically
- Attend to precision
- Look for and make use of structure
- Look for and express regularity in repeated reasoning.

# Appendix B: Mathematical skills and processes

# Descriptors of Process Standards in WNCP's *Common Curriculum Framework for Grades 10-12 Mathematics* (WNCP, 2008, p. 6)

Students are expected to:

- use *communication* in order to learn and express their understanding
- make *connections* among mathematical ideas, other concepts in mathematics, everyday experiences and other disciplines
- demonstrate fluency with *mental mathematics and estimation*
- develop and apply new mathematical knowledge through problem solving
- develop mathematical *reasoning*
- select and use *technology* as a tool for learning and solving problems
- develop *visualization* skills to assist in processing information, making connections and solving problems.

# Descriptors of Process Standards in NCTM's *Principles and Standards for School Mathematics* (NCTM, 2000, p. 402)

- 1. Analyzing a problem
  - Build new mathematical knowledge through problem solving
  - Apply and adapt a variety of appropriate strategies to solve problems
  - Monitor and reflect on the process of mathematical problem solving

#### 2. Reasoning and Proof

- Make and investigate mathematical conjectures
- Evaluate mathematical arguments and proofs
- Use various types of reasoning and methods of proof

## 3. Communication

- Organize and consolidate one's mathematical thinking
- Analyze and evaluate the mathematical thinking and strategies of others
- Use the language of mathematics to express mathematical ideas precisely

## 4. Connections

- Recognize and use connections among mathematical ideas
- Seek to understand how mathematical ideas interconnect
- Seek to build a coherent network of ideas

# 5. Representation

- Use representations to organize, record, and communicate mathematical ideas
- Select, apply, and translate among mathematical representations to solve problems
- Use representation to model and interpret physical, social, and mathematical phenomenon

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Pages 111 through 116 redacted for the following reasons: S3

# At a Glance: Provincial curricular interpretations of the WNCP

Province	Summary	
Overall observations	<ul> <li>Most of the rationale, goals, nature of mathematics, mathematics processes, and curriculum organizers seem directly influenced by the WNCP documents: K- 9 (2006) and 10-12 (2008).</li> </ul>	
	<ul> <li>Learning standards (outcomes) are usually connected to the mathematical processes</li> <li>Additional Calculus courses do not connect to current curriculums</li> </ul>	
	Additional Calculus courses do not connect to current curriculums	
BC	<ul> <li>K-7, 8-9 have slightly different goals and rationales than the 10-12</li> <li>K-9 – one pathway (naming: Math 8)</li> <li>Follows the WNCP 10-12 pathways without changes (including naming)</li> </ul>	
	For K. 7 and 8.9	
	<ul> <li>Nature of mathematics: change, constancy, number sense, patterns, relationships, spatial sense, and uncertainty.</li> </ul>	
	<ul> <li>Mathematical processes: communication; connections; mental mathematics and estimation; problem solving; reasoning; technology; and visualization</li> <li>Curriculum organizers: number; patterns and relations; shape and space; statistics and probability (for all grades, although some grades do not have learning outcomes)</li> </ul>	
	<ul> <li>For WNCP 10-12:</li> <li>Mathematical processes and nature of mathematics: same as K-7, 8-9</li> <li>Curriculum organizers: different for each pathway, and different at each grade level</li> </ul>	
	Additional course: Calculus 12 (2000)	
	*For Adult Education: Acct 11 will satisfy graduation component	
AB	<ul> <li>Currently redesigning curriculum</li> <li>Two main pathways, separating at the grade 8 level: regular and Knowledge and Employability (KE)</li> <li>K-12 (regular) curriculum share the same rationale and goals (K-9, 10-12)</li> <li>KE courses share the same rationale and set of goals (KE 8-9, KE 10-4, 20-4)</li> <li>Pathways</li> <li>K-7 – one pathway (naming: Math 7)</li> </ul>	
	<ul> <li>Math 8 and 9 - two pathways: 1) regular (Math 9), and 2) KE (KE Math 9)</li> <li>Math 10 - three pathways: 1) Foundations of Math and Pre-Calculus 10 (Math 10 C); 2) Apprenticeship and Workplace 10 (Math 10-3); and 3) KE (Math 10-4)</li> <li>Math 11 - four pathways: 1) Pre-Calculus (Math 20-1); 2) Foundations of Math (Math 20-2); 3) Apprenticeship and Workplace Math (Math 20-3); and 4) KE (Math 20-4)</li> <li>Math 12 - three pathways: 1) Pre-Calculus (Math 30-1); 2) Foundations of Math (Math 30-2); and 3) Apprenticeship and Workplace Math (Math 30-3);</li> </ul>	

	<ul> <li>For K-9 (regular): <ul> <li>Nature of mathematics: change, constancy, number sense, patterns, relationships, spatial sense, and uncertainty.</li> <li>Mathematical processes: communication; connections; mental mathematics and estimation; problem solving; reasoning; technology; and visualization</li> <li>Strands (Curriculum organizers): number; patterns and relations; shape and space; statistics and probability (for all grades, although some grades do not have learning outcomes)</li> </ul> </li> <li>For Math 10-12 (regular): <ul> <li>Mathematical processes and nature of mathematics: same as K-7, 8-9</li> <li>Curriculum organizers: different for each pathway, and different at each grade level</li> <li>Note: Some of the outcomes for Mathematics 20-2 and 30-2 in this program of studies are different from the outcomes for Foundations of Mathematics in the Common Curriculum Framework (1)</li> </ul> </li> <li>For KE 8-9 and KE 10-4, 20-4: <ul> <li>Promote cross-curricular, community and workplace connections.</li> <li>Mathematical processes, and Strands: same as K-9 (regular)</li> <li>Nature of mathematics: change, constancy, dimension (size and scale), number, pattern, quantity, relationships, shape, and uncertainty</li> <li>Problem-solving framework for KE Mathematics is included</li> </ul> </li> </ul>
	Additional course: Mathematics 31 (1995)
SK	<ul> <li>Currently revising curriculum</li> <li>Rationale and goals differ from WNCP</li> <li>Four goals: logical thinking, number sense, spatial sense, and mathematical attitude (Mathematics as a Human Endeavour)</li> <li>Broad areas of learning: 1) building a disposition for learning; 2) building a sense of self and community; and 3) building engaged citizens</li> <li>Cross-curricular competencies: 1) constructing knowledge; 2) identity and interdependence; 3) developing literacy; and 4) social responsibility</li> <li>Cross-curricular connections to other subjects are provided and clarified</li> <li>K-9 – one pathway (naming, Math 8)</li> <li>10-12 – three pathways (same naming as WNCP)</li> </ul>
	<ul> <li>For K-12 (overall): <ul> <li>Strands (Curriculum organizers): number; patterns and relations; shape and space; statistics and probability (for all grades, although some grades do not have learning outcomes)</li> <li>Mathematical processes: communication; connections; mental mathematics and estimation; problem solving; reasoning; technology; and visualization</li> <li>Nature of mathematics: not included or mentioned</li> </ul> </li> <li>For 10-12:</li> </ul>

	• Even though the K-12 (overall) document states the strands (curriculum
	organizers), they are not used for the 10-12. Outcomes are connected to the
	broad goals (on the print version), then stated and numbered.
	Additional courses: A30, B30, C30 (1996) *Calculus courses
MB	<ul> <li>Curriculum is a revision of the WNCP Math K-9 (2006) and 10-12 (2008) documents</li> </ul>
	<ul> <li>Rationale and goals are the same for K-8 and 9-12</li> </ul>
	<ul> <li>K-8 – one pathway (naming: Math 8)</li> </ul>
	<ul> <li>Math 9 – two options: 1) Math 9 (10 Foundations); and 2) Math 9 Transitional Mathematics</li> </ul>
	<ul> <li>*Math 9 Transitional Mathematics is not intended to replace Math 9 (10F)</li> </ul>
	• Math 10-12 – three pathways: Introduction to Applied and Pre-Calculus
	Mathematics 20S (Foundations of Mathematics and Pre-Calculus 10), Essential
	Mathematics 20S, 30S, 40S (Apprenticeship and Workplace), Applied
	Mathematics 30S, 40S (Foundations of Mathematics), and Pre-Calculus 30S, 40S
	For K-8:
	<ul> <li>Nature of mathematics: change, constancy, number sense, patterns,</li> </ul>
	relationships, spatial sense, and uncertainty.
	Mathematical processes: communication; connections; mental mathematics and
	estimation; problem solving; reasoning; technology; and visualization
	<ul> <li>Strands (Curriculum organizers): number; patterns and relations; shape and</li> </ul>
	space; statistics and probability (for all grades, although some grades do not
	have learning outcomes)
	For 9-12.
	Math 9 Transitional Mathematics and each grade of Essential Mathematics is a
	one-credit course consisting of two half-credits, each emphasizing consumer
	applications, problem solving, decision making and spatial sense.
	Curriculum organizers, mathematical processes, and nature of mathematics:
	same as K-8
	Some of the outcomes have been altered or deleted
	Additional Courses: Accounting 30S, 40S
	*No Calculus course
ΥT	Follows BC
NT	Follows AB
NU	Follows AB

# **Aboriginal Math Resources**

The following document is bibliography of resources related to Aboriginal math, which were searched for in ERIC during October, 2009. The search string that was used was: [Aboriginal OR Indigenous OR "First Nations" OR "American Indian" or "Native American"] AND [math OR mathematics]. Those results that seemed most relevant were included in this document.

The resources have been organized under eight different categories: Theory and pedagogy, Curriculum development, Teacher perceptions, Teacher education, Culturally responsive case studies, Student attitudes toward mathematics, Standards, guidelines and recommendations, and Statistics. Although, many of the below resources fit into multiple categories, they are only listed under the heading which seemed to fit best. Each "entry includes the full references, and the author supplied abstract.

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January 29, 2013

**VIA EMAIL** Ref: 165137

Susan Lambert, President BC Teachers Federation 100 - 550 West 6th Avenue Vancouver BC V5Z 4P2

Dear Susan Lambert:

As you know, the province has been engaged in discussions and consultations to transform education in British Columbia (see: <u>Enabling Innovation: Transforming Curriculum and</u> <u>Assessment</u>).

During the summer and fall of 2012, teams of educators and academics met to experiment with the curriculum structure in a variety of subject areas. Also during this time, researchers developed draft working definitions of the cross-curricular competencies.

Attached are two documents that outline this work to date on both curriculum design and crosscurricular competency definitions. These documents will be posted on the Ministry of Education website at the end of this week.

We would appreciate you sharing these documents with the presidents of your Provincial Specialist Associations. In the very near future, we will also be contacting you regarding designating teacher representatives to subsequent curriculum development work.

We look forward to your feedback and your contributions to BC's curriculum development process.

Sincerely,

Runth

Rod Allen Superintendent of Learning Learning Division

Attachment

Ministry of Education Learning Division

Mailing Address: PO Box 9887 Stn Prov Govt Victoria BC V8W 9T6 Location: 620 Superior St Victoria BC

> EDU-2014-00033 Page 125

Pages 126 through 175 redacted for the following reasons:

#### Notes

#### NCTM: http://www.nctm.org/Default.aspx

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#### Favorites list (24/02/14)

http://www.nationalnumeracy.org.uk/what-is-numeracy/index.html

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http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/CBS\_AskingEffectiveQuestions.pdf

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Pages 180 through 192 redacted for the following reasons: S3

From:	Cormode, Sarah EDUC:EX
То:	Horner, Melissa EDUC:EX
Cc:	Colgate, Elaine EDUC: EX
Subject:	RE: Thank you!
Date:	Tuesday, April 9, 2013 9:32:46 PM

Thank you so much for giving me time to meet your team today!

I was meeting tonight with a few people and Nella Nelson (Principal of Aboriginal Education) was speaking highly of the mathematics resources developed in Alaska, so I thought I'd share them with you in case they are helpful with your group. The Haida Nation will have their resources for sale apparently. Joanne Yovanovich is the contact person there.

http://www.ankn.uaf.edu/npe/ansme.html

http://www.haidanation.ca/Pages/Haida\_Legends/Lessons/RavenLesK-3.pdf

I'll send along the posters in a moment also.

Have a very successful day tomorrow! Great to see you again! Welcome, almost back!

Sarah

Not Responsive
Not Responsive

Hi Melissa,

It is good to hear from you. I am intrigued by what you are doing and I like that you are really making an effort to incorporate indigenous perspectives in math. I guess my one wondering – and it is always a wondering – is how to do this authentically so it does not feel like an add on. I'm attaching a couple other pieces you might find relevant. The CMESG 2010 piece is where you will find the proceedings from the work I did with Ed and Dawn that you asked about. The second piece is an article that just came out in MERJ, and the third piece is from discussions about Indigenous Perspectives in Mathematics from a CMEF meeting in 2009.

It is a little difficult for me to give you feedback without having a better perspective on what you are doing. If you wanted to Skype me into a discussion sometime, I would be happy to do that. Just let me know.

Lisa

From: <Horner>, "Melissa EDUC:EX" <<u>Melissa.Horner@gov.bc.ca</u>> Date: Tuesday, 11 June, 2013 7:08 PM To: STFX STFX <<u>lborden@stfx.ca</u>> Subject: Hello!

Hello Lisa!

I hope you had a wonderful time in Victoria, and a safe return home.

It was wonderful to be able to sit with you, and to hear your story first-hand. The article I had read of yours was, "The 'Verbification' of mathematics: Using the grammatical structures of Mi'kmaq to support student learning" (not sure where it was originally published). One of your references in that text was a piece you wrote with Drs Doolittle and Wiseman, would you happen to have an e-copy? I'd love to read it and can't seem to get my hands on a copy.

I am attaching slides from a meeting we had in May; I'm not sure how much time you have available, but I'd love to quickly pick your brain on a few things. The whole presentation is interesting, but slides 13-16 are most relevant to our work. We are really trying to infuse the First Peoples Principles of Learning into/through our curriculum.

Our content is still in a very drafty stage, so I can't currently share too much, but I will share a little:

Area of Learning:	Grade: 8	
Mathematics		
Big Ideas:	·	
if !supportLists? · endif? proportional reasoning helps one n	nake sense of how	
quantities are related in real-life contexts		
if !supportLists? < ![endif]>understanding whole number multiplication and division		
helps one make sense of these operations with fractions and integers		
if !supportLists? endif? linear relations can be represented in many ways, and these		
representations have important connections		
if !supportLists? · endif? one can make sense of 3-D objects	through different	
perspectives, including nets and views		

if !supportLists? ·	$ data collection and representation helps us to communicate$
with others.	

Content
Students will know and understand:
<pre><?if !supportLists?>` <?endif?>logic and patterns to solve games and puzzles <?if !supportLists?>` <?endif?>multiplication and division of decimals, fractions, mixed numbers, and integers <?if !supportLists?>` <?endif?>rates and proportional reasoning <?if !supportLists?>` <?endif?>squares, cubes and exponents <?if !supportLists?>` <?endif?>factors, prime factors, and numerical radicals <?if !supportLists?>` <?endif?>two- variable linear relations extended to four quadrants, including graphing <?if !supportLists?>` <?endif?>surface area and volume of prisms and pyramids <?if !supportLists?>` <?endif?>construction, views and numes of 3D objects <?if !supportLists?>` <?endif?>data collection, display, and analysis, including surveying and sampling <?if !supportLists?>` <?endif?>data collection, display, and analysis, including surveying and sampling <?if !supportLists?>` <?endif?></pre>

As you can see, right above this sentence, the First Peoples Principles of Learning is currently explicitly living in the skills and processes (renamed curricular competencies).

In your opinion, is this a best fit? More information will be in the front matter, but my teacher team wanted, and rightly so, something right on the content.

If you are too busy to provide a reply, I understand!

Cheers,

Melissa Horner

Curriculum and Assessment Ministry of Education Ph: 778-679-7596 C: Melissa.Horner@gov.bc.ca Pages 198 through 199 redacted for the following reasons: S3

<b>D</b>			
Province	Guiding Concepts	Curriculum Organizers	Specific Outcomes
BC	Nature of	Measurement	18
	Mathematics:	Algebra and Number	
Alberta	change, consistency,	Relations and Functions	
*Called 10C	number sense,		
(combined)	patterns,		
Manitoba	relationships, spatial		19
*Called Grade 10	sense, and		
Introduction to	uncertainty.		
Applied and Pre-	Mathematical		
Calculus	Processes:		
Mathematics	communication;		
	connections; mental		
	mathematics and		
	estimation; problem		
	solving; reasoning;		
	technology; and		
	visualization.		
			10
Saskatchewan	K-12 Mathematic	Not using curriculum organizers.	10
<b>.</b>	Broad Goals: logical	Fundamentian forms the commission	
Cross-Curricular	thinking, number	I Explanation from the curriculum.	
Commetensies		The content of K 12 methometics can be executed in a variaty of your in the	
Competencies	sense, spatial sense,	The content of K-12 mathematics can be organized in a variety of ways. In the	
Competencies (embedded):	sense, spatial sense, and Mathematics as a	The content of K-12 mathematics can be organized in a variety of ways. In the grades 10-12 curricula, the outcomes are not grouped according to strands (as in	
Competencies (embedded): Thinking, Identity	sense, spatial sense, and Mathematics as a human endeavour	The content of K-12 mathematics can be organized in a variety of ways. In the grades 10-12 curricula, the outcomes are not grouped according to strands (as in the elementary mathematics curricula) or by topic (as in past curricula). The primary reasons for this are: a suscinct set of high loyal outcomes for each grade	
Competencies (embedded): Thinking, Identity and	sense, spatial sense, and Mathematics as a human endeavour	The content of K-12 mathematics can be organized in a variety of ways. In the grades 10-12 curricula, the outcomes are not grouped according to strands (as in the elementary mathematics curricula) or by topic (as in past curricula). The primary reasons for this are: a succinct set of high level outcomes for each grade, and variation between grades and pathways in terms of the topics and content.	
Competencies (embedded): Thinking, Identity and Interdependence, Literacios, and	sense, spatial sense, and Mathematics as a human endeavour Mathematical Processor:	The content of K-12 mathematics can be organized in a variety of ways. In the grades 10-12 curricula, the outcomes are not grouped according to strands (as in the elementary mathematics curricula) or by topic (as in past curricula). The primary reasons for this are: a succinct set of high level outcomes for each grade, and variation between grades and pathways in terms of the topics and content within different courses. For eace of reference, the outcomes in this curriculum are	
Competencies (embedded): Thinking, Identity and Interdependence, Literacies, and Social	sense, spatial sense, and Mathematics as a human endeavour Mathematical Processes:	The content of K-12 mathematics can be organized in a variety of ways. In the grades 10-12 curricula, the outcomes are not grouped according to strands (as in the elementary mathematics curricula) or by topic (as in past curricula). The primary reasons for this are: a succinct set of high level outcomes for each grade, and variation between grades and pathways in terms of the topics and content within different courses. For ease of reference, the outcomes in this curriculum are numbered using the following system: EP10 #, where # is the number of the	
Competencies (embedded): Thinking, Identity and Interdependence, Literacies, and Social Responsibility	sense, spatial sense, and Mathematics as a human endeavour Mathematical Processes: communication; connections: mental	The content of K-12 mathematics can be organized in a variety of ways. In the grades 10-12 curricula, the outcomes are not grouped according to strands (as in the elementary mathematics curricula) or by topic (as in past curricula). The primary reasons for this are: a succinct set of high level outcomes for each grade, and variation between grades and pathways in terms of the topics and content within different courses. For ease of reference, the outcomes in this curriculum are numbered using the following system: FP10.#, where # is the number of the outcomes in the list of outcomes. It should be noted for example, that EP10.1 need	
Competencies (embedded): Thinking, Identity and Interdependence, Literacies, and Social Responsibility	sense, spatial sense, and Mathematics as a human endeavour Mathematical Processes: communication; connections; mental mathematics and	The content of K-12 mathematics can be organized in a variety of ways. In the grades 10-12 curricula, the outcomes are not grouped according to strands (as in the elementary mathematics curricula) or by topic (as in past curricula). The primary reasons for this are: a succinct set of high level outcomes for each grade, and variation between grades and pathways in terms of the topics and content within different courses. For ease of reference, the outcomes in this curriculum are numbered using the following system: FP10.#, where # is the number of the outcome in the list of outcomes. It should be noted, for example, that FP10.1 need not be taught before EP10.10. Teachers are encouraged to design learning	
Competencies (embedded): Thinking, Identity and Interdependence, Literacies, and Social Responsibility	sense, spatial sense, and Mathematics as a human endeavour Mathematical Processes: communication; connections; mental mathematics and estimation: problem	The content of K-12 mathematics can be organized in a variety of ways. In the grades 10-12 curricula, the outcomes are not grouped according to strands (as in the elementary mathematics curricula) or by topic (as in past curricula). The primary reasons for this are: a succinct set of high level outcomes for each grade, and variation between grades and pathways in terms of the topics and content within different courses. For ease of reference, the outcomes in this curriculum are numbered using the following system: FP10.#, where # is the number of the outcome in the list of outcomes. It should be noted, for example, that FP10.1 need not be taught before FP10.10. Teachers are encouraged to design learning activities that integrate outcomes from throughout the curriculum so that students.	
Competencies (embedded): Thinking, Identity and Interdependence, Literacies, and Social Responsibility	sense, spatial sense, and Mathematics as a human endeavour Mathematical Processes: communication; connections; mental mathematics and estimation; problem solving: reasoning;	The content of K-12 mathematics can be organized in a variety of ways. In the grades 10-12 curricula, the outcomes are not grouped according to strands (as in the elementary mathematics curricula) or by topic (as in past curricula). The primary reasons for this are: a succinct set of high level outcomes for each grade, and variation between grades and pathways in terms of the topics and content within different courses. For ease of reference, the outcomes in this curriculum are numbered using the following system: FP10.#, where # is the number of the outcome in the list of outcomes. It should be noted, for example, that FP10.1 need not be taught before FP10.10. Teachers are encouraged to design learning activities that integrate outcomes from throughout the curriculum so that students develop a comprehensive and connected view of mathematics rather than viewing	
Competencies (embedded): Thinking, Identity and Interdependence, Literacies, and Social Responsibility	sense, spatial sense, and Mathematics as a human endeavour Mathematical Processes: communication; connections; mental mathematics and estimation; problem solving; reasoning; technology; and	The content of K-12 mathematics can be organized in a variety of ways. In the grades 10-12 curricula, the outcomes are not grouped according to strands (as in the elementary mathematics curricula) or by topic (as in past curricula). The primary reasons for this are: a succinct set of high level outcomes for each grade, and variation between grades and pathways in terms of the topics and content within different courses. For ease of reference, the outcomes in this curriculum are numbered using the following system: FP10.#, where # is the number of the outcome in the list of outcomes. It should be noted, for example, that FP10.1 need not be taught before FP10.10. Teachers are encouraged to design learning activities that integrate outcomes from throughout the curriculum so that students develop a comprehensive and connected view of mathematics rather than viewing mathematics as a set of compartmentalized ideas and separate topics. The	
Competencies (embedded): Thinking, Identity and Interdependence, Literacies, and Social Responsibility	sense, spatial sense, and Mathematics as a human endeavour Mathematical Processes: communication; connections; mental mathematics and estimation; problem solving; reasoning; technology; and visualization	The content of K-12 mathematics can be organized in a variety of ways. In the grades 10-12 curricula, the outcomes are not grouped according to strands (as in the elementary mathematics curricula) or by topic (as in past curricula). The primary reasons for this are: a succinct set of high level outcomes for each grade, and variation between grades and pathways in terms of the topics and content within different courses. For ease of reference, the outcomes in this curriculum are numbered using the following system: FP10.#, where # is the number of the outcome in the list of outcomes. It should be noted, for example, that FP10.1 need not be taught before FP10.10. Teachers are encouraged to design learning activities that integrate outcomes from throughout the curriculum so that students develop a comprehensive and connected view of mathematics rather than viewing mathematics as a set of compartmentalized ideas and separate topics. The ordering and grouping of the outcomes in Foundations of Mathematics and Pre-	
<b>Competencies</b> (embedded): Thinking, Identity and Interdependence, Literacies, and Social Responsibility	sense, spatial sense, and Mathematics as a human endeavour Mathematical Processes: communication; connections; mental mathematics and estimation; problem solving; reasoning; technology; and visualization.	The content of K-12 mathematics can be organized in a variety of ways. In the grades 10-12 curricula, the outcomes are not grouped according to strands (as in the elementary mathematics curricula) or by topic (as in past curricula). The primary reasons for this are: a succinct set of high level outcomes for each grade, and variation between grades and pathways in terms of the topics and content within different courses. For ease of reference, the outcomes in this curriculum are numbered using the following system: FP10.#, where # is the number of the outcome in the list of outcomes. It should be noted, for example, that FP10.1 need not be taught before FP10.10. Teachers are encouraged to design learning activities that integrate outcomes from throughout the curriculum so that students develop a comprehensive and connected view of mathematics rather than viewing mathematics as a set of compartmentalized ideas and separate topics. The ordering and grouping of the outcomes in Foundations of Mathematics and Precalculus 10 is at the discretion of the teacher.	
Competencies (embedded): Thinking, Identity and Interdependence, Literacies, and Social Responsibility	sense, spatial sense, and Mathematics as a human endeavour Mathematical Processes: communication; connections; mental mathematics and estimation; problem solving; reasoning; technology; and visualization.	The content of K-12 mathematics can be organized in a variety of ways. In the grades 10-12 curricula, the outcomes are not grouped according to strands (as in the elementary mathematics curricula) or by topic (as in past curricula). The primary reasons for this are: a succinct set of high level outcomes for each grade, and variation between grades and pathways in terms of the topics and content within different courses. For ease of reference, the outcomes in this curriculum are numbered using the following system: FP10.#, where # is the number of the outcome in the list of outcomes. It should be noted, for example, that FP10.1 need not be taught before FP10.10. Teachers are encouraged to design learning activities that integrate outcomes from throughout the curriculum so that students develop a comprehensive and connected view of mathematics rather than viewing mathematics as a set of compartmentalized ideas and separate topics. The ordering and grouping of the outcomes in Foundations of Mathematics and Precalculus 10 is at the discretion of the teacher. https://www.edonline.sk.ca/webapps/moe-curriculum-	

calculus 10.xml			
		calculus 10.xml	

Pages 202 through 232 redacted for the following reasons: S3 To check out: Improving numeracy in Canada (2000) http://en.copian.ca/library/research/nls/inpub/numeracy/improve.pdf

Great piece: What might 'numeracy' in the 21<sup>st</sup> C mean? Canadian, 2000. <u>http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.135.371&rep=rep1&type=pdf</u>

Ontario press release -<u>http://news.ontario.ca/edu/en/2014/01/new-math-supports-and-resources-for-the-classroom.html?utm\_source=ondemand&utm\_medium=email&utm\_campaign=p</u>

Education for Early Mathematical Literacy: More than Maths Know-How: (Queensland, 2000) <u>http://www.merga.net.au/documents/RP742005.pdf</u>

Teaching numeracy: helping children become confident mathematicians: http://books.google.ca/books?hl=en&lr=&id=MUKPkT\_VPjYC&oi=fnd&pg=PA185&dq=nume racy+is+the+ability+to+process,+interpret,+and+communicate+%2B+Evans&ots=2x2eisHEAa &sig=dm3BqeeDMCsfYs-i0ZQ\_teE-S6Y#v=onepage&q&f=false

http://www.tandfonline.com/doi/abs/10.1080/14926150309556584

Mathematics and democracy: The case for quantitative literacy. NCED, 2001.

De Lange, J. (2003). Mathematics for literacy. *Quantitative literacy: Why numeracy matters for schools and colleges*, 80.

Scotland's numeracy page

http://education.alberta.ca/department/ipr/inclusion/capacity/numeracyforallpilot.aspx

- "Mathematics is a common human activity, increasing in importance in a rapidly advancing, technological society. A greater proficiency in using mathematics increases the opportunities available to individuals. Students need to become mathematically literate in order to explore problem-solving situations, accommodate changing conditions, and actively create new knowledge in striving for self-fulfillment." (Alberta Learning, 1996, p.2)
- 2. "Mathematics is more than numbers just as reading is more that letters. Literacy involves placing numbers into meaningful context in daily living." (Balas, 1997)

Balas, Andrea K. "The Mathematics and Reading Connection. ERIC Digest." (1997).

 "Like literacy, numeracy is not a case of one's either being proficient or not, rather individuals' skills are "situated along a continuum of different purposes and levels of accomplishment with numbers. Numeracy includes a range of skills that are necessary for initial survival in a new country and for functioning as a fully literate person." (Ciancone, 1996)

Ciancone, T. (1996). *Numeracy in the adult ESL classroom*. National Clearinghouse for ESL Literacy Education.

- 4. "the ability to think and express oneself effectively in quantitative terms" (The American Heritage Dictionary of the English Language).
- 5. The Merriam-Webster's Online Dictionary defines numeracy as "the capacity for quantitative thought and expression."
- 6. "Numeracy not only incorporates the individual's abilities to use and apply mathematical skills efficiently and critically, but also requires the person to be able to interpret and communicate about mathematical information and reasoning processes."(Ontario Literacy Coalition, 2001)

http://www.essentialskillsontario.ca/

7. "... numeracy is more about the ability to use and apply rather than just knowing." (Hughes, Desforges, Mitchell, & Carre, 2000)

Hughes, M. (2000). *Numeracy and beyond*. C. Desforges, & C. Mitchell (Eds.). Buckingham: Open University Press.

8. "Numeracy is the ability to process, interpret and communicate numerical, quantitative, spatial, statistical, even mathematical, information, in ways that are appropriate for a variety of contexts, and that will enable a typical member of the culture or subculture to participate effectively in activities that they value." (Evans, 2000)

Evans, J., & Tsatsaroni, A. (2000). Mathematics and its publics: Texts, contexts and users. *Social epistemology*, *14*(1), 55-68.

- 9. "... something about number and computation." (report on research by the Mathematics Council of the Alberta Teachers' Association (2004))
- 10. "... numeracy tends to be more commonly used in the United Kingdom and Europe while the term mathematical literacy is more commonly used in North America." (Hoogland (2003), citing Jablonka, 2009)

Hoogland, K. (2003). Mathematical literacy and numeracy. 2008-03-30]. http://www. gecijferdheid. nl/pdf/Ho-oglandJablonka\_UK. PD FI.

11. "Unlike most other jurisdictions in Canada, British Columbia has used the term numeracy rather than mathematical literacy." (Holdfast Report, 2004)

https://www.wncp.ca/media/39083/final\_report.pdf

12. "describe students' capacity to use their mathematical knowledge for informed citizenship." In this way mathematical literacy is "an individual's capacity to use mathematics as a fully functioning member of a society." (Ball & Stacey, 2003)

Ball, L., & Stacey, K. (2001). New literacies for mathematics: A new view of solving equations. *The Mathematics Educator*, *6*(1), 55-62.

- 13. "The mathematics necessary to be an informed voter." (anonymous)
- 14. "Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded mathematical judgments and to engage in mathematics, in ways that meet the needs of that individual's current and future life as a constructive, concerned and reflective citizen." (Organization for Economic Co-Operation and Development, 2003b)

http://www.oecd.org/pisa/pisaproducts/PISA%202012%20framework%20e-book final.pdf

15. "... students' capacities to analyse, reason, and communicate ideas effectively by posing, formulating and solving mathematical problems in a variety of domains and situations." (Organization for Economic Co-Operation and Development, 2003a)

http://www.oecd.org/pisa/pisaproducts/PISA%202012%20framework%20e-book final.pdf

16. "Mathematical Literacy, should enable the learner to become a self-managing person, a contributing worker and a participating citizen in a developing democracy. Mathematical Literacy will ensure a broadening of the education of the learner which is suited to the modern world." (Department of Education (South Africa), 2003)

http://www.education.gov.za/LinkClick.aspx?fileticket=25TyZ4RTmKc%3D&tabid=246&mid=594

- 17. "An aggregate of skills, knowledge, beliefs, dispositions, habits of mind, communication abilities, and problem solving skills that people need in order to engage effectively in quantitative situations arising in life and work. " (from the International Life Skills Survey, Center for Educational Statistics of Statistics Canada, 2001)
- 18. "Mathematical Literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to use, apply, analyse, process, interpret and communicate mathematical information, and solve problems involving that information in ways that meet the needs of the individual as a constructive, concerned, reflective and involved citizen." (Holdfast Report, 2004)
- 19. "Logical thinking, analysis of evidence, and statistical reasoning are far more important for engaged citizenship in the twenty-first century, than traditional algebraic and mathematical skills. The new literacy, from this perspective, is really about reasoning more than arithmetic: assessing claims, detecting fallacies, evaluating risks, weighing evidence." Steen (2000)

Steen, L. A. (2001). Mathematics and numeracy: Two literacies, one language. *The Mathematics Educator*, *6*(1), 10-16.

20. "... mathematical knowledge put to functional use in a multitude of different contexts and a variety of ways that call for reflection and insight. Of course, for such use to be possible and

viable, a great deal of fundamental mathematical knowledge and skills (as often taught in schools) are needed. In the linguistic sense, reading literacy cannot be reduced to, but certainly presupposes, a wide vocabulary and a substantial knowledge of grammatical rules, phonetics, orthography, and so forth. In the same way, mathematical literacy cannot be reduced to, but presupposes, knowledge of mathematical terminology, facts, and procedures, as well as skills in performing certain operations, and carrying out certain methods." (Organization for Economic Co-Operation and Development, 2003b)

- 21. "We believe that application is at the heart of numeracy. At all levels of learning mathematics pupils need to use and build on what they already know in order to progress further." (Hughes, Desforges, Mitchell, & Carre, 2000)
- 22. "... to develop in young people the ability to acquire specialized knowledge and skills as and when they need them during the course of their working lives." (Devlin, 2000)

Devlin, K. J. (2000). *The math gene: How mathematical thinking evolved and why numbers are like gossip.* New York: Basic Books.

Devlin, K. (2000). *The language of mathematics: making the invisible visible*. Macmillan.

23. "It is important to note that math is not simply about numbers and numerical operations. Mathematics knowledge and skills also include the ability to draw inferences, see relationships and reason logically." (National Research Council 1989)

http://www.nrc-cnrc.gc.ca/eng/index.html?PHPSESSID=697bb52f80246fabe2c5ea50419c6a1f

24. "The outcome of this focus should be numerate pupils who are confident enough to tackle mathematical problems without going immediately to teachers or friends for help" (Department for Education and Employment – United Kingdom, 1999).

25. "Better numeracy standards occur when teachers:

• Structure their mathematics lessons and maintain a good pace;

- Provide daily oral and mental work to develop and secure pupils' calculation strategies and rapid recall skills;
- Devote a high proportion of lesson time to direct teaching of whole classes and groups, making judicious use of textbooks, worksheets and ICT resources to support teaching, not to replace it;
- Demonstrate, explain and illustrate mathematical ideas, making links between different topics in mathematics and between mathematics and other subjects;
- Use and give pupils access to number lines and other resources, including ICT, to model mathematical ideas and methods;
- Use and expect pupils to use correct mathematical vocabulary and notation;
- Question pupils effectively, including as many of them as possible, giving them time to think before answering, targeting individuals to take account of their attainment and needs, asking them to demonstrate and explain their methods and reasoning, and exploring reasons for any wrong answers;
- Involve pupils and maintain their interest through appropriately demanding work, including some non-routine problems that require them to think for themselves; and,
- Ensure that differentiation is manageable and centred around work common to all the pupils in a class, with targeted, positive support to help those who have difficulties with mathematics to keep up with their peers." (Department for Education and Employment United Kingdom, 1999).
- 26. "In keeping with a wide range of jurisdictions across North America and around the world it would appear best to use the term mathematical literacy rather than numeracy, especially in light of the Alberta survey results, which found that most people defined numeracy as *something about number and computation.*" (Holdfast Report, 2004)

- 27. "Steen (2000) identifies a range of goals for numeracy within five different dimensions. These dimensions are:
  - Practical for immediate use in the routine tasks of life;
  - Civic to understand major public policy issues;
  - Professional to provide skills necessary for employment;
  - Recreational to appreciate games, sports, lotteries;
  - Cultural as part of the tapestry of civilization.

Any definition of mathematical literacy probably needs to consider the above dimensions along with the following:

- The ability to think and express oneself effectively in quantitative terms;
- The capacity for quantitative thought and expression;
- The ability to use and apply mathematical skills efficiently and critically;
- The capacity to use their mathematical knowledge for informed citizenship and as a constructive, concerned and reflective member of society;
- The capacity to identify and understand the role that mathematics plays in the world;
- The ability to make well-founded mathematical judgements;
- The ability to engage in mathematics, in ways that meet the needs of that individual's current and future life; The ability to analyse, reason, and communicate ideas effectively by posing, formulating and solving mathematical problems related to a variety of situations;
- The capacity to be able to interpret and communicate about mathematical information and reasoning processes;
- The aggregate of mathematical skills, knowledge, dispositions, habits of mind, and problem solving skills in combination with the communication abilities that people need in order to engage effectively in quantitative situations; and
- The recognition of the technological nature of our world." (Holdfast Report, 2004)
- 28. "Mathematical Literacy contributes to the attainment of the Critical and Developmental Outcomes in that it enables learners to:
  - Use mathematical process skills to identify, pose and solve problems creatively and critically;
  - Work collaboratively in teams and groups to enhance mathematical understanding;
  - Organise, interpret and manage authentic activities in substantial mathematical ways that demonstrate responsibility and sensitivity to personal and broader societal concerns;
  - Collect, analyse and organize quantitative data to evaluate and critique conclusions;
  - Communicate appropriately by using descriptions in words, graphs, symbols, tables and diagrams;
  - Use mathematical literacy in a critical and effective manner to ensure that science and technology are applied responsibly to the environment and to the health of others;
  - Demonstrate that a knowledge of mathematics assists in understanding the interrelatedness of systems and how they affect each other;
  - Be prepared to use a variety of individual and co-operative strategies in learning;
  - Engage responsibly with quantitative arguments relating to local, national and global issues;

- Be sensitive to the aesthetic value of mathematics;
- Explore the importance of mathematical literacy for career opportunities;
- Realise that mathematical literacy contributes to entrepreneurial success.

Learners working toward Mathematical Literacy should be able to:

- Use numbers with understanding to solve real-life problems in different contexts including the social, personal and financial;
- Use mathematically-acquired skills to perform with understanding financially-related calculations involving personal, provincial and national budgets;
- Model relevant situations using suitable functions and graphical representation to solve related problems;
- Describe, represent and analyse shape and space in two dimensions and three dimensions using geometrical skills;
- Engage critically with the handling of data (statistics and chance), especially in the manner in which these are encountered in the media and in presenting arguments;
- Use computational tools competently (a scientific calculator is taken as the minimum)." (Holdfast Report, 2004)
- 29. "... empowers people by giving them tools to think for themselves, to ask intelligent questions of experts, and to confront authority confidently. These are the skills required to thrive in the modern world." (The Quantitative Literacy Design Team, 2001)

http://www.maa.org/sites/default/files/pdf/QL/MathAndDemocracy.pdf

- 30. "... quantitative literacy involves mathematics acting in the world. Typical numeracy challenges involve real data and uncertain procedures but require primarily elementary mathematics. The test of numeracy, as of any literacy, is whether a person naturally uses appropriate skills in many different contexts." (The Quantitative Literacy Design Team, 2001)
- 31. "... numeracy is not just one among many subjects but an integral part of all subjects." ((The Quantitative Literacy Design Team, 2001)
- 32. "Numerate: imply *at homeness*, which is the ability to make use of mathematical skills which enables an individual to cope with the practical demands of everyday life." (The Quantitative Literacy Design Team, 2001)

- 33. "... an ability to have some appreciation and understanding of information which is presented in mathematical terms." (The Quantitative Literacy Design Team, 2001)
- 34. "Quantitative literacy is the knowledge and skills required to apply arithmetic operations, either alone or sequentially, using numbers embedded in printed material (e.g. balancing a checkbook, completing an order form)." (The Quantitative Literacy Design Team, 2001)
- 35. "Quantitative literacy: an aggregate of skills, knowledge, beliefs, dispositions, habits of mind, communication capabilities, and problem solving skills that people need in order to engage effectively in quantitative situations arising in life and work." (The Quantitative Literacy Design Team, 2001)
- 36. "The elements of quantitative literacy:
  - Confidence with mathematics be comfortable with quantitative ideas and at ease in applying quantitative methods ... the opposite of "math anxiety"; it makes numeracy as natural as ordinary language.
  - Cultural appreciation understand the nature and history of mathematics, its role in scientific inquiry and technological progress, and its importance for comprehending issues in the public realm.
  - Interpreting data to be able to [reason] with data, [read] graphs, [draw] inferences, and [recognize] sources of error. This perspective focuses on the data rather than the formulas or relationships.
  - Logical thinking to be able to [analyze] evidence, [reason] carefully, [understand] arguments, [question] assumptions, [detect] fallacies, and [evaluate] risks. These individuals "accept little at face value; they constantly look beneath the surface, demanding appropriate information to get at the essence of issues.
  - Making decisions to be able to [use] mathematics to make decisions and solve problems in everyday life. Mathematics is not something done only in mathematics class but a powerful tool for living, as useful and ingrained as reading and speaking.
  - Mathematics in context to be able to [use] mathematical tools in specific settings where the context provides meaning.
  - Number sense [to have] accurate intuition about the meaning of numbers, confidence in estimation, and common sense in employing numbers as a measure of things.
  - Practical skills [to know] how to solve quantitative problems that a person is likely to encounter at home or at work. [These individuals] are adept at using elementary mathematics in a wide variety of common situations.

- Prerequisite knowledge [to have] the ability to use a wide range of algebraic, geometric, and statistical tools that are required in many fields of postsecondary education.
- Symbol sense [to be] comfortable using algebraic symbols and at ease in reading and interpreting them, and exhibiting good sense about the syntax and grammar of mathematical symbols." (The Quantitative Literacy Design Team, 2001)
- 37. "Mathematics literacy: an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded mathematical judgements and to engage in mathematics in ways that meet the needs of that individual's current and future life as a constructive, concerned and reflective citizen." (The Quantitative Literacy Design Team, 2001)
- 38. "Mathematics in action." (The Quantitative Literacy Design Team, 2001)
- 39. "To possess skills such as arithmetic skills (simple mental arithmetics, estimating arithmetic calculations, etc.), interpreting data and draw inferences from data, computer skills (using spreadsheets, recording data, extrapolating data, etc.), modeling skills (to formulate problems, seek patterns, draw conclusions, etc.), statistics skills (understand variability, correlation, causation, etc.), understand probability (understand "coincidences", evaluate risks, etc.), and reasoning skills (make inferences, check hypotheses, make generalizations, etc.)." (The Quantitative Literacy Design Team, 2001)
- 40. "Numeracy has no special content of its own, but inherits its content from its context." (The Quantitative Literacy Design Team, 2001)
- 41. "In teaching quantitative literacy, content is inseparable from pedagogy and context is inseparable from content." (The Quantitative Literacy Design Team, 2001)
- 42. "... quantitative literacy is not only about arithmetic and higher mathematics but also about a general skill (or habit of mind) that is required in many subjects across the curriculum." (Cohen, 2001)

Cohen, P. C. (2001, December). Democracy and the numerate citizen: Quantitative literacy in historical perspective. In *Proceedings of the National Forum on Quantitative Literacy held at the National Academy of Sciences, Washington, DC*.

43. "The concept of quantitative literacy is rooted in the connection between mathematics and reason. ... quantitative literacy is tied to two rather different concepts – numeracy and reason." (Richards, 2001)

Richards, J. L. (2001). Connecting mathematics with reason. *MATHEMATICS and DEMOCRACY*, 31.

44. "Literacy implies an integrated ability to function seamlessly within a given community of practice. ... it is profoundly social, and is therefore a moving target because its contents depend on a particular social context" (Ewell, 2001)

Ewell, P. T., & Jones, D. P. (2006). State-level accountability for higher education: On the edge of a transformation. *New Directions for Higher Education*, *2006*(135), 9-16.

- 45. "... literacies are for the most part practiced invisibly and subconsciously by members of a community, not pulled out selectively and applied deliberately to a particular set of circumstances. In practicing quantitative literacy, therefore, we would expect that an individual would not consciously say "Oh, this is mathematics" and enter a different ("learned") way of thinking and acting. Instead, he or she would simply act competently without invoking a disciplinary context at all." (Ewell, 2001)
- 46. "Quantitative literacy is presumably practiced together with other literacies in most actual circumstances, whereas mathematics as a discipline can be practiced on its own." (Ewell, 2001)
- 47. " as a literacy, quantitative literacy is not practiced in isolation nor can it be separated from a particular social context." (Ewell, p.48)
- 48. "... the predilection and ability to make use of various modes of mathematics thought and knowledge to make sense of situations we encounter as we make our way through the world." (Schoenfeld, 2001)

Schoenfeld, A. H. (2000). Purposes and methods of research in mathematics education. *Notices of the AMS*, *47*(6), 641-649.

49. "... the ability to understand and reason with numerical information. That ability enables people to be comfortable with numerical data and to use them in meaningful ways, in particular to make well-reasoned decision." (Manaster, 2001)

Manaster, A. B. (2001). Mathematics and numeracy: Mutual reinforcement. Mathematics and democracy: The case for quantitative literacy. Princeton, NJ: The National Council on Education and the Disciplines.

- 50. "... [with quantitative literacy], numbers are descriptors of characteristics of the objects being studied, in mathematics numbers themselves are the objects of study" (Manaster, 2001)
- 51. "Students should see their mathematical skills as tools that are important to use on a daily basis. They should be as comfortable with the language of mathematics as with the language of English. The threads of mathematics should be present and exploited in every course in the high school curriculum. Logical thinking is as important in history and social science as it is in mathematics and science." (Stith, 2001)
- 52. "When students can see the connection between school mathematics and the mathematics of real life." (Stith, 2001)
- 53. "Without quantitative literacy, people cannot fully understand what is in the everyday news, what is in everyday life." (Usiskin, 2001)

Usiskin, Z. (2001). Quantitative literacy for the next generation. *Mathematics and Democracy*.

54. "... more mathematics and science with not automatically lead to numeracy. Quantitative illiteracy cannot be overcome by introducing more subject matter." (Cuban, 2001)

Cuban, L. (2001). Encouraging progressive pedagogy. *Mathematics and Democracy*, 87-92.

55. "... mathematics focuses on climbing the ladder of abstraction, while quantitative literacy clings to context. Mathematics asks students to rise above context, while quantitative literacy asks students to stay in context. Mathematics is about general principles that can be applied in a range of contexts; quantitative literacy is about seeing every context through a quantitative lense." (Hughes-Hallett, 2001)

Hughes-Hallett, D. (2001). Achieving numeracy: The challenge of implementation. *Mathematics and Democracy*, 93-98.

- 56. "the ability to identify quantitative relationships in a range of contexts" (Hughes-Hallett, 2001)
- 57. "involves insight as well as algorithms [..] involves reflection, judgment, and above all, experience." (Hughes-Hallett, 2001)
- 58. "... efforts to intensify attention to the traditional mathematics curriculum do not necessarily lead to increased competency with quantitative data and numbers. While perhaps surprising to many in the public, this conclusion follows from a simple recognition—that is, unlike mathematics, numeracy does not so much lead upwards in an ascending pursuit of abstraction as it moves outward toward an ever richer engagement with life's diverse contexts and situations." (Orrill, 2001)

Orrill, R. (2001). Mathematics, numeracy, and democracy. *LA Steen, Mathematics and democracy: The case for quantitative literacy.* 

- 59. "Numeracy can be defined as the combination of mathematical knowledge, problem solving and communication skills required by all persons to function successfully within our technological world." (BCAMT)
- "Numeracy is the willingness and ability to apply and communicate mathematical understanding and procedures in novel and meaningful problem solving situations." (BC SD43)

- 61. "Numeracy is not only an awareness that mathematical knowledge and understandings can be used to interpret, communicate, analyze, and solve a variety of novel problem solving situations, but also a willingness and ability to do so." (BC SD 57)
- 62. "Numeracy refers to the application of mathematical understanding in daily activities at school, at home, at work, and in the community. It involves both using mathematical skills and knowing how mathematics can be used to solve problems. Just as there is more to literacy than teaching the rules and procedures of language, there is more to numeracy than teaching the rules and procedures of mathematics. Numerate individuals not only "know" mathematics, but understand it in personally meaningful terms. They feel competent and confident about their ability to draw on the necessary knowledge and apply it in new and relevant ways." (BC Performance Standards, 2002)
- 63. "Numeracy is a proficiency which is developed mainly in mathematics but also in other subjects. It is more than an ability to do basic arithmetic. It involves developing confidence and competence with numbers and measures. It requires understanding of the number system, a repertoire of mathematical techniques, and an inclination and ability to solve quantitative or spatial problems in a range of contexts. Numeracy also demands understanding of the ways in which data are gathered by counting and measuring, and presented in graphs, diagrams, charts and tables." (UK Standards)
- 64. "Being numerate is about having the disposition and competence to use mathematics to solve practical problems outside mathematics and as a tool for learning beyond the mathematics classroom." (Australian Curriculum Framework)
- 65. "Numeracy is the knowledge and skills required to effectively manage and respond to the mathematical demands of diverse situations." (International Adult Literacy and Skills Survey)
- 66. "Numeracy is the mathematics for effective functioning in one's group and community, and the capacity to use these skills to further one's own development and of one's community." (Beazley, 1984).

- 67. "Numeracy involves abilities that include interpreting, applying and communicating mathematical information in commonly encountered situations to enable full, critical and effective participation in a wide range of life roles." (Queensland Department of Education, 1994)
- 68. "To be numerate is more than being able to manipulate numbers, or even being able to 'succeed' in school or university mathematics. Numeracy is a critical awareness which builds bridges between mathematics and the real-world, with all its diversity." (Johnston, 1994).
- 69. "Numerate behaviour is observed when people manage a situation or solve a problem in a real context; it involves responding to information about mathematical ideas that may be represented in a range of ways; it requires the activation of a range of enabling knowledge, factors and processes." (ALL Numeracy Team, 2002)
- 70. "Numeracy is fluency with math facts." (BC SDxx)
- 71. "What is numeracy?
  - "At homeness" with numbers
  - Appreciation of mathematics
  - Confidence in math
  - Reasoning skills
  - Mental math ability
  - Use symbols
  - Sense of numbers
  - Use mathematical models
  - Interpret data
  - Read and interpret graphs
  - Likewise, numeracy is more than knowing about numbers and number operations...it's more than being able to solve for "x"...So we also wouldn't say that a student who can graph a line or solve an equation is necessarily numerate." (Association for Career and Technical Training)

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### **Numeracy in British Columbia**

The following information outlines British Columbia's approach to developing and maintaining numeracy excellence, from Kindergarten to graduation, and beyond. Numeracy is the application of mathematical knowledge, problem solving ability, and communication skills that an educated citizen needs to function successfully within our world. Numeracy encompasses a positive attitude toward mathematics and a capacity to identify and understand the role that mathematics plays in the world. Numerate citizens can make well-founded judgments and confidently use and engage with mathematics in ways that meet their needs. In the school curriculum, numeracy proficiency is developed mainly in Mathematics lessons but is also applied and extended in other subjects.

Educators, parents, students, and the greater public in British Columbia have expressed a need for greater emphasis on financial literacy in the K-12 educational program; this coincides with an increased focus from around the globe. Financial literacy is the knowledge and understanding of financial concepts, and the skills, motivation, and confidence to apply this knowledge and understanding in making effective and accurate decisions across a range of financial contexts.

Graduation requirement consultations with educators, parents, and the greater public in British Columbia have indicated a need for numeracy standards that students would need to successfully complete for graduation.

Grades	In the classroom	Provincial Assessments	National Assessments
Grades K-9	In the classroom <i>Current (WNCP) and draft mathematics</i> <i>curriculum(K-9)</i> Numeracy is stated as a key element of the rationale for teaching Mathematics. It is also core to one of the components of mathematics, Number Sense, which is emphasized throughout the current and transformed curriculum. The realigned curriculum contains financial literacy concepts and content from grades 1 to 9. <i>BC Early Numeracy Project (K-1)</i> The project's purpose was to develop assessment and instructional tools to support early numeracy development, particularly for children at risk. Three classroom resources, <i>Assessing Early</i> <i>Numeracy, Supporting Early Numeracy,</i> <i>Whole Group Follow-up</i> and a parent resource <i>Math for Families – Supporting</i> <i>Numeracy at Home</i> , are the result. <i>BC Barfarmanca Standards</i> (1, 8) for	Provincial Assessments Foundation Skills Assessment (4 and 7) The assessment is developed by B.C. educators to measure learning skills linked to the provincial curriculum and performance standards. Results help the Province, school districts and schools keep track of how well our education system is doing and identify learning areas that may need improvement. The numeracy assessment measures math problem- solving skills that students have gained during several years of learning. With this assessment BC has been	National Assessments Pan-Canadian Assessment Program (8) The Pan-Canadian Assessment Program (PCAP) is a national Grade 8 assessment developed by the provincial and territorial ministries of education, through the Council of Ministers of Education, Canada (CMEC). Mathematics is one of three areas assessed by PCAP. PCAP takes place every three years and is an important measure of in-school success at secondary school.
	BC Performance Standards(1-8)for Numeracy The BC performance standards have been developed for voluntary use in BC schools.	assessment, BC has been focusing on numeracy performance since 1999.	

	They describe the professional judgments of a significant number of BC educators about standards and expectations. The standards focus exclusively on performance assessment. In performance assessment students are asked to apply the skills and concepts they have learned to complete complex, realistic tasks.		
10	Under the WNCP framework, students in grade 10 have two choices for their mathematics 10 courses: 1) Foundations of Mathematics and Pre-Calculus 10; and 2) Apprenticeship and Workplace Mathematics 10. Both of these courses place a strong emphasis on the Number Sense.	Mathematics 10 exams Regardless of which, or both, mathematics 10 course students have chosen, they must complete the relevant mathematics 10 provincial exam. The exam is worth 20% of the class mark and is required for graduation. Number sense is the focus for 30% of each exam.	Programme for International Student Assessment (PISA) PISA is a collaborative effort among member countries of the Organisation for Economic Co- operation and Development (OECD). It measures 15 year old students' abilities in science, math, and reading and takes place every three years. PISA is an important measure in secondary school and provides policy- related international indicators of student performance.
11-12	Students in grades 11 and 12 have three pathways to choose from: Pre-Calculus, Foundations of Mathematics, and/or Apprenticeship and Workplace. All three courses are comprised of a Number Sense component. Students may also take Calculus 12, in addition to other numeracy related courses available in other Areas of Learning, such as Accounting (Applied Skills). Students must complete a minimum of two mathematics courses in order to graduate.		
Adult Education	Students must have at least one mathematics 11 or 12 course in order to obtain their Adult Education graduation; Accounting 11, 12 and Financial Accounting 12 also satisfy this requirement.		

# Table 4.13 Multiple conceptions of curriculum integration

(This table combines Québec's conceptions of curriculum integration with those found earlier in the research literature as a potential conceptual framework for other jurisdictions attempting to define their conceptions of curriculum integration.)

If the intention is	with ideals based on	Then curriculum might be designed with
to		opportunities for students to
Integrate	Relevance;	Explore and resolve issues-based
changing societal	Conscious and critical	problems;
issues	reflective practices, actions;	Attempt complex tasks where there are
	Authentic learning	multiple possible solutions
Integrate fields of	Flexible boundaries;	Explore broad disciplinary fields;
knowledge	Reduced fragmentation;	Define core learnings from disciplines;
(disciplines)	Avoiding redundancy;	Use disciplines to explore issues from
	Recognizing disciplinary	many angles
	contributions to learning	
Integrate	Knowledge in service of	Ask questions, gather, assess, process,
fundamental	inquiry and decision-making;	and transform information for a variety of
learning	Lifelong learning;	purposes and in a variety of contexts
processes	Mastery of generic skills	(e.g., cross-curricular and subject specific
		competencies)
Integrate	Metacognitive skills;	Be actively involved in negotiation of
knowledge	Multiple intelligences;	meaning;
(deeper levels of	Higher-order thinking skills;	Assess understanding in context rather
meaning-making)	Constructivism (connected	than in isolation (authentic assessment);
	and contextualized learnings	Use self-evaluations, portfolios, journals;
	improve retention)	Choose multiple presentation formats
Integrate citizens	Democratic principles;	Collaborate and work in groups;
into society	Rights and responsibilities;	Voice opinions, respect the views of
	Collaboration;	others and negotiate collective actions
	Social cohesion	
Integrate cultural	Transmission of collective	Explore historical connections from
experiences and	prior knowledge, values, and	diverse perspectives to better understand
heritage	understandings	personal links with cultural identities
Integrate	All learners have the	Incorporate prior learning experiences;
personal	potential to succeed;	Differentiate learning according to needs,
experiences and	Holistic development;	interests, and questions;
Identity	Active participation	Initiate, adapt, and structure activities
Integrate skills	Applied and life skills;	Interact with contribute to communities;
for community	Skills for employment and	Access community resources
living	community contributions	
Integrate global	Multiple perspectives;	Access multiple sources of information;
and pluralistic	Globalization and change;	Study interrelationships between culture,
perspectives		Values, and learning
	Equilable access;	Differentiate learning strategies,
learners into an	Fair treatment of all	evaluation methods, resources, and
equitable system	Individuals and groups	educational paths to achieve success.
Integrate	Efficient and effective	Have input into, negotiate, experiment,
operational	systemic and delivery	adapi, and evaluate models for
models, Ideology,	models that serve learners	curriculum design, implementation, and
and discourse	and stakenoiders.	assessment; Challenge traditional or deminant modules
		Challenge traditional or dominant models;
		Use a common vocabulary and ideology

# **Mathematics**

Cross-Jurisdictional Review Executive Summary

Jurisdictions included:

- Australia
- British Columbia
- Finland
- Ontario
- Saskatchewan

Also completed: an examination of Provincial Interpretations of the WNCP, covering:

- Alberta
- British Columbia
- Saskatchewan
- Manitoba
- Yukon
- Nunavut
- Northwest Territories

Trends:

- Many jurisdictions and provinces are in a process of changing their curriculum
- From the western provinces, only BC is following the WNCP as it is directly presented. Other provinces have made changes adding, deleting, and/or changing learning standards
- Curriculum changes are reflecting a need for content that is broad and deep, with fewer learning standards
- Many provinces have a separate Calculus course, which is based on a much older version of their curriculum
- One pathway is typically offered for K-7; increasing the number of pathways as the grade level increases
- The typical number of pathways for 10-12 is four
- Specific attempts in the curriculum to improve students' attitudes towards Mathematics
- Not using curriculum organizers to separate learning standards
- Respecting the needs of all learners and incorporating visuals and manipulatives in the teaching and student practice

K-12 Structures:

• Pre-Kindergarten is not typically included

- Some jurisdictions are separating the Kindergarten curriculum from the 1-12; within the K curriculum, Mathematics is emphasized as learning connected to the other subjects (see Ontario as an example)
- Most jurisdictions use curriculum organizers that span across K-10
- Some jurisdictions use concepts such as Nature of Mathematics or Mathematical Processes to help shape and/or organize their learning standards. These are typically the same for grades K-12.
- Most jurisdictions have some method of organizing their learning standards; usually by curriculum organizer, sub-organizer, and then numbered standards.

# Goals/Rationale:

- Many jurisdictions have included the idea of 'the educated citizen' in their rationale.
- Alignment across the grades seems to also be a goal for many revisions that are currently underway, along with using cross-curricular objectives and content.

### Competencies:

- Most jurisdictions that do not have a recent curriculum revision do not make competencies transparent.
- Several jurisdictions refer to "cross-curricular themes" or "essential graduation learnings" but these read as competencies.

Exemplary example: Saskatchewan (K-10)

- Includes 4 cross-curricular competencies: Developing Thinking, Developing Identity and Interdependence, Developing Literacies, and Developing Social Responsibility
- The possible connections between the competencies and cross-curricular connections are provided.
- Saskatchewan's website is also very easy to navigate; however, there are some discrepancies (connections to broad goals/competencies) between the online curriculum and the print version.

#### Mathematics Curriculum research base

The mathematics teacher and curriculum team strived to create curriculum that:

- empowers students and teachers
- aligns with current and promising mathematics teaching and learning practices
- is developmentally appropriate
- focuses on higher level concepts and content
- is structured through problem solving and inquiry
- connects to mathematical habits of mind and practices

The team also aimed to incorporate First Peoples' content, practices, and principles of learning into each of the curriculum documents and materials.

The mathematics curriculum was influenced by such work as:

# Balcean, P.L., & Klassen, W. (2008). *Teaching critical mathematics thinking: Mathematical mindedness*. Retrieved from http://ocs.sfu.ca/fedcan/index.php/csse/csse2008/paper/view/437/308

Balcean and Klassen argue that the mathematics teaching context provides a powerful opportunity to enable students to think critically while better engaging students in thinking about mathematics. However, within the literature various understandings of mathematical thinking and notions of what constitutes critical thinking are used interchangeably, leading to both conceptual and pedagogical confusion. They offer the conception of "critically thoughtful mathematical thinking" (CTMT) and a supporting pedagogy below that addresses the goal of teaching critical thinking and aspects of what some identify as mathematical thinking.

Bishop, A.J. (1991). *Mathematical enculturation: A cultural perspective on mathematics education*. Norwell, MA: Kluwer Academic Publishers.

Bishop's book explores mathematics as 'a way of knowing', while taking a cultural look at the subject and analyzing the educational consequences of the cultural perspectives. The author explores a range of anthropological, cross-cultural, and historical literature concerning mathematics and culture. Bishop's aim is to create a new conception of mathematics which both recognizes and demonstrates its relationship with culture: the notion of mathematics as a cultural product, the environmental and societal activities which stimulate mathematical concepts, and the cultural values which mathematics embodies.

Boaler, J. (2002). Learning from teaching: Exploring the relationship between reform curriculum and equity. *Journal for Research in Mathematics Education*, *33*(4), 239-258.

Boaler's paper considers concerns that have been raised about the potential of reform-oriented curriculum for promoting equity and argues that investigations into equitable teaching must pay attention to the particular practices of teaching and learning that are enacted in classrooms. Data are presented from two studies in which middle and high school teachers using reform-oriented

mathematics curriculum achieved a reduction in linguistic, ethnic, and class inequalities in their schools. The teaching and learning practices that were employed are shown to be central to the attainment of equity, suggesting that relational analysis of equity, that go beyond the curriculum, to include the teacher and their teaching, are critical.

Charles, R.I. (2005) Big ideas and understandings as the foundation for elementary and middle school mathematics. *Journal of Mathematics Education Leadership, 7*(3).

The author's purpose of this paper is to initiate a conversation about the notion of Big Ideas in mathematics. Although Big Ideas have been talked about for some time, Charles states that they have not become part of mainstream conversations about mathematics standards, curriculum, teaching, learning, and assessment. A definition of a Big Idea is presented here along with a discussion of their importance. Then a set of Big Ideas and Understandings for elementary and middle school mathematics is proposed. The paper closes with some suggestions for ways Big Ideas can be used.

Common Core State Standards Initiative (2010). *Common core state standards in mathematics*. Retrieved from http://www.corestandards.org/Math/Practice

The Standards for Mathematical Practice developed by the Common Core State Standards Initiative describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

Cuoco, A., Goldenberg, E. P., & Mark, J. (1996). Habits of mind: An organizing principle for a mathematics curriculum. *Journal of Mathematical Behavior*, *15*(4), 375-402.

The authors maintain that much more important than specific mathematical results are the habits of mind used by the people who create those results, and they describe elements of a curriculum that elevates the methods by which mathematics is created, the techniques used by researchers, to a status equal to that enjoyed by the results of that research. Their goal is not to train large numbers of high school students to be university mathematicians, but rather to allow high school students to become comfortable with ill-posed and fuzzy problems, to see the benefit of systematizing and abstraction, and to look for and develop new ways of describing situations. The authors state that while it is necessary to infuse courses and curricula with modern content, what's even more important is to give students the tools they'll need to use, understand, and even make mathematics that doesn't yet exist. A curriculum organized around habits of mind tries to close the gap between what the users and makers of mathematics do and what they say.

First Nations Education Steering Committee (FNESC). (2011). *Teaching mathematics in a First Peoples context: Grades 8 and 9*. Retrieved from http://www.fnesc.ca/wordpress/wp-content/uploads/2011/03/Aug-12-2011-MFP-8-+-9-web.pdf

This Guide is designed to help teachers of Mathematics 8 and 9 in British Columbia extend their existing practice to incorporate new approaches that make the BC school system more reflective of the realities of First Peoples in BC, and improve overall levels of student success when it comes to meeting provincially prescribed standards for Mathematics at these grade levels. The Guide has been developed by FNESC with assistance from the British Columbia Ministry of Education and support from the Education Partnerships Program of Indian and Northern Affairs Canada.

Goldenberg, E. P., Shteingold, N., & Feurzeig, N. (2003). Mathematical habits of mind for young children. In F. K. Lester & R. I. Charles (Eds.), *Teaching mathematics through problem solving: Prekindergarten-Grade 6* (pp. 15-29). Reston, VA: National Council of Teachers of Mathematics.

The main goal of the 'teaching mathematics through problem solving' approach is to help students develop a deep understanding of mathematical concepts and methods by engaging them in problematic tasks in which the mathematics to be learned is embedded. This book aims to provide the coherence and direction teachers need to use problem solving to teach mathematics.

Levasseur, K., & Cuoco, A. (2003). Mathematical habits of mind. In H. L. Schoen (Ed.), *Teaching mathematics through problem solving: Grade 6-12* (pp. 23-37). Reston, VA: National Council of Teachers of Mathematics.

This book is about problem solving as a means for acquiring new mathematical knowledge, rather than as an activity to be engaged in after studying various concepts and skills. Articles by a variety of authors are grouped into four sections: 'Issues and Perspectives' deals with the conceptual and historical background of teaching mathematics through problem solving; 'Tasks and Tools for Teaching and Learning' looks at selecting and using appropriate tasks and learning tools so that the intended mathematical understanding will result; 'In the Classroom' focuses on how teaching mathematics through problem solving might play out in the classroom; and a final chapter presents a research perspective. Incorporated throughout is a collection of 'Teacher Stories', in which individual teachers share their own experiences of teaching mathematics through problem solving. The chapter and book look at the big questions and provides teachers with practical ideas to use in the classroom.

Lipka, J., Sharp, N., Brenner, B., Yanez, E., & Sharp, F. (2005). The relevance of culturally based curriculum and instruction: The case of Nancy Sharp. *Journal of American Indian Education, 44*(3), 31-54.

The authors illustrate Ms. Sharp's case to show how this experienced Yup'ik teacher steeped in the traditions of her culture effectively implemented a culturally based math module. Ms. Sharp's pedagogical creativity allowed her to authentically bring together a core academic content area, math, with Yup'ik traditions, knowledge, and ways of relating. This case shows through systematic micro-ethnography, interview data, and "insider" analysis that when Ms. Sharp used expert-apprentice

modeling, joint productive activity, and cognitive apprenticeship. Her students were attentive, highly focused on the math task, and learned about symmetry, congruence, and patterns. Expert-apprentice modeling usually associated with "crafts" and usually dismissed as an ineffective classroom pedagogical tool was a key ingredient for Ms. Sharp's success. One project outcome measures her students performed well when compared to other treatment classes that used this module and to the control classes. The case shows how curriculum based on aspects of indigenous culture, combined with effective pedagogical practices derived from the community and accommodated to the culture of schooling results in appreciable student learning.

Lunney Borden, L. (2011). The 'verbification' of mathematics: Using the grammatical structures of Mi'kmaq to support student learning. *For the learning of mathematics, 31*(3), 8-13.

As part of a larger project focused on transforming mathematics education for Aboriginal students in Atlantic Canada, this paper reports on the role of the Mi'kmaw language in mathematics teaching. Examining how mathematical concepts are described in Mi'kmaq gives insight into ways of thinking. Shifting classroom discussions to reflect Mi'kmaw verb-based grammar structures, referred to as 'verbification', is described through the example of a grade 3 lesson on prisms and pyramids. 'Verbification' shows tremendous promise as a way to support Mi'kmaw learners as they negotiate their space between school-based mathematics and their own cultural ways of knowing and doing mathematics.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Principles and Standards for School Mathematics are guidelines produced by the National Council of Teachers of Mathematics in 2000, setting forth recommendations for mathematics educators.

Niemi, D., Vallone, J., & Vendlinkski, T. (2006). *The power of big ideas in mathematics education: Development and pilot testing of POWERSOURCE© assessments* (No. 697). CRESST Tech. Rep. Retrieved from https://www.cse.ucla.edu/products/reports/R697.pdf

The authors state the characteristics of expert knowledge—interconnectedness, understanding, and ability to transfer—are inextricably linked, a point that is critically important for educators and constitutes a major theme of this paper. In this paper they explore how an analysis of the architecture of expert knowledge can inform the development of assessments to help teachers move students toward greater expertise in mathematics, and they present examples of such assessments. The authors also review student responses and preliminary results from pilot tests of assessments administered in sixth-grade classes in a large urban school district. Their preliminary analyses suggest that an assessment strategy based on the structure of mathematical knowledge can reveal deficiencies in student understanding of and ability to apply fundamental concepts of pre-algebra, and has the potential to help teachers remediate those deficiencies.

Organization for Economic Co-operation and Development (OECD). (2013). *PISA 2012 Assessment and analytical framework: Mathematics, reading, science, problem solving and financial literacy.* Retrieved from http://www.oecd-ilibrary.org/education/pisa-2012-assessment-and-analyticalframework\_9789264190511-en;jsessionid=2rirr8k1fe166.x-oecd-live-02.

PISA 2012 Assessment and Analytical Framework presents the conceptual framework underlying the fifth cycle of PISA. Similar to the previous cycles, the 2012 assessment covers reading, mathematics and science, with the major focus on mathematical literacy. Two other domains are evaluated: problem solving and financial literacy. Students respond to a background questionnaire and, as an option, to an educational career questionnaire as well as another questionnaire about Information and Communication Technologies (ICTs). Additional supporting information is gathered from the school authorities through the school questionnaire and from the parents through a third optional questionnaire. Sixty-six countries and economies, including all 34 OECD member countries, are taking part in the PISA 2012 assessment.

# Small, M. (2009). *Big ideas from Dr. Small: Creating a comfort zone for teaching mathematics*. Scarborough, ON: Nelson Canada.

Big Ideas from Dr. Small aims to provide mathematics teachers with what they need to know to teach the curriculum while focusing on the big ideas for each concept. Each book includes hundreds of practical activities and follow-up questions to use in the classroom. The accompanying Facilitator's Guide could be used by coaches and leaders of Professional Learning Communities.

Van de Walle, J., Karp, K. S., & Bay-Williams, J. M. (2012). *Elementary and middle school mathematics: Teaching developmentally*. Boston, MA: Allyn & Bacon.

Elementary and Middle School Mathematics is intended primarily for pre-service and practicing elementary and junior high school mathematics teachers in Canada. Elementary and Middle School Mathematics provides ideas and discussion to help student-teachers develop a real understanding of the mathematics they will teach. This text aims to show the benefits of constructivist - or student centered - mathematics instruction. The authors maintain that it provides a strong theoretical perspective on children's learning of mathematics - not just a casual overview.

Western and Northern Canadian Protocol (WNCP). (2011). *Common Curriculum Framework for Grades K-12 Mathematics*. Retrieved from http://www.wncp.ca/english/subjectarea/mathematics/ccf.aspx

WNCP Mathematics includes a Common Curriculum Framework for both Kindergarten to Grade 9 and Grades 10–12 along with authorized resources and information for publishers. The Common Curriculum Framework for K–9 Mathematics was developed by the seven ministries of education in collaboration with teachers, administrators, parents, business representatives, post-secondary educators and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and achievement indicators agreed upon by the seven jurisdictions.

# K-12 Mathematics Ancillary Materials Overview Spring 2013 Mathematics Curriculum Development Working Group

# Preface

During the course of the three meetings that took place in the spring of 2013, the mathematics working group, composed of teachers, district-level educators, and Dr. Wendy Klassen, felt it necessary to create supporting documents and express key ideas to support and extend the curriculum development deliverables.

- 1) K-7 developmental continuum
- 2) Grade band curriculum
- 3) Elaborations
- 4) Delivery model options for 8-12

In addition to this document, there is also a separate piece that discusses the BC mathematics curriculum from within a First Peoples culture, heritage, and tradition.

#### 1) K-7 developmental continuum

\*Draft, currently in the process of revision Filename: K-7 DevCon Draft 1

The K-7 developmental continuum (DevCon) was the foundation on which the K-7 Learning Standards were created upon. Although there are similarities to a standard Scope and Sequence, the DevCon reflects a student-centered, constructivist, developmental approach and teaching, learning, and assessment.

The working team considered:

- the key learning for each grade level in relation to the previous and next year's
- the progression of the key learning
- the developmental appropriateness of the learning for the student.

Colour coding:

- heading and sub-heading: yellow
- K-2: green
- 3-5: orange
- 6-7: light blue
- extension: dark blue.

The DevCon provides powerful information for the teacher, parent, and student. The teacher can use the DevCon to help diagnose current level, to plan teaching, and to assess learning. The parent and student can use it to help match where the student currently is, and to see how the learning progresses.

The working team used the curriculum strands of: Number; Pattern and Relations; Statistics and Probability; and Shape and Space to help organize the content. These strands are the conceptual lenses of the Big Ideas for each grade. The sub-headings in the first column (yellow) and the content (Victoria curriculum refers to these as, "Indicators of Progress") throughout were informed by a number of sources, including: the Western and Northern Canadian Protocol (WNCP); the National Council of Teachers of Mathematics (NCTM); documents from the Surrey School District (#36); and the work of Dr. Marion Small.

Although the current document goes only to grade 7, there is a place for this work to continue up to and including grade 10.

On a global scale, using this type of continuum has been done; notably, in New South Wales (Australia), Northern Ireland, and Victoria (Australia).

http://www.curriculumsupport.education.nsw.gov.au/secondary/mathematics/assets/pdf/continuum/k 10 continuum.pdf

http://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/mathscontinuum/indicatorsgrid.p df

http://misabarrie.curriculum.org/storage/281/files2/ContinuumBasedMath.pdf

http://www.nicurriculum.org.uk/docs/assessment/ACCS Training/foundation/DS Using Maths.pdf

# 2) Grade band curriculum

\*Draft, in need of some revision Filename: K-2 by grade band sample draft 1

Building from the use of the DevCon, the working team also asked for some consideration for a curriculum that focussed on learning standards that were organized around grade bands, and not by individual grades. They felt that this would be a better reflection of how students actual learn, and that it fit in with the interactive online model that the Ministry is proposing.

Under this model, teachers with a specific grade band would be able to 'click' on the content under the learning standard and locate on the developmental continuum where the student is.
### 3) Elaborations

\*In need of development

Regardless of whether or not the curriculum was presented by individual grades or by grade bands, the working group felt strongly about the need for multiple 'layers' of content underneath the one-page of learning standards.

The layers would need to meet the needs of the many different types of mathematics teachers: the novice, the experienced teacher, the generalist, the specialist, and more. The creation and presentation of this material is a daunting but essential task.

The elaborations could provide further detail on all of the information on the one-page curriculum document, but it also could be an avenue for ideas/examples for teaching, learning, and assessment. The use of an online platform for this type of work is an ideal fit, as it can provide multiple pathways to information in a way that would be more of a challenge if using a paper medium.

Examples of similar websites/applications can be found in Saskatchewan and Australia.

https://www.edonline.sk.ca/webapps/moe-curriculum-BBLEARN/index.jsp?lang=en

# http://www.australiancurriculum.edu.au/Mathematics/Rationale

Elaborations, in a number of forms, are also being considered and developed in other subject areas, such as Social Studies and Science, within the BC curriculum development teams.

# 4) Delivery model options for 8-12

\*Draft, in need of revision Filenames: Math 8-10 core draft 1 Math 8-10 choice draft 1 Math core-choice flowchart draft 1 Math optional topics draft 1

In thinking about making changes to the content, the working group (in particular, the 8-10 team) was concerned that changing the content was not enough – that there is a need to change the teaching practice around the content, and to change the way in which the students were able to make choices about the content.

They felt that in this time of curriculum development change that there was also room to make even bigger changes to ways in which the students received the content. The main concern or ambition was to provide greater flexibility for both the teacher and the student. The 8-10 team presented two main options:

- A) Core/Choice
- B) Optional topics

### A) Core/Choice

In this model, there are two sets of courses (other names: modules, topics, units of study):

- 1) Core: the key mathematic learning that the Educated Citizen needed
- 2) Choice: the mathematic courses that students would need for their own future career or life choices.

In the words of the 8-10 teachers:

### Core

"Mathematics is integrated in everyday life and used to make sense of our lived experience. The Educated Citizen is capable of using math to critically analyze information presented to them in their work, finances and daily life. In order to do so they must be able to use math strategies to make informed choices. Engaging in the fundamentals of mathematics allows the educated citizen to communicate effectively and identify their role in a global society. Through inquiry of a problem, people who have flexible fluency can explore multiple solutions in order to solve a problem in context. Having all graduates complete the Core units identified below will allow further progression and discovery in the field. Flexible fluency with math topics will aid students in their ability to creatively envision solutions or construct their point of view. The capacity to perform a variety of topics comes with time and effort for all students, but should not be considered a hindrance to those who are easily capable, nor those who need extra time to learn a topic. To that end the Core is designed so that students may progress at their own pace while being overseen in a classroom as described in the suggested topic/unit based model. The model is also designed to promote self confidence and engagement in mathematics by making it more manageable in size and duration, in addition to stressing the 21st Century skills needed in today's reality as identified by the cross curricular competencies. "

### Choice

"The purpose of the Choice topics/units is to increase the exposure of the Educated Citizen to the variety of mathematical fields, while preparing them for possible further studies. Mathematics can be explored in its multiple forms dependant on the interest of the student. Engaging in the application and theory of mathematics allows students to differentiate between possible strategies to choose the one best suited for the situation and themselves. As mathematics increases in complexity, students discover developing patterns that interconnect. As mathematical learning progresses towards the abstract or symbolic, reasoning and analysis is used to critically evaluate the reasonableness of a designed process. Within the described model, with guidance, students are able to design a learning plan that will fit future needs and abilities. Learning plans can be flexible, with opportunities for personalized learning and cross-curricular applications."

From grades 8-10, students would have a set number of Core units that they would need to progress through, in addition to a number of Choice courses that they would select from.

The team created a flowchart to help illustrate the Core/Choice delivery model, with related content material, in addition to the individual grade content that was required from the deliverables.

S13

# Terminology:

Tracking: students are placed into classes based on their marks, not necessarily by ability or interest. Typically, advanced tracks lead to post-secondary, and lower tracks to vocational skills. It is usually difficult for students to move from one track to another. Strong tracking systems can be traced to English-speaking countries, and this system is losing popularity within the global perspective.

Streaming or Differentiating: supporters of the 'tracking' system seem to refer to it as streaming or differentiating. Critics of the system call it tracking, which has the more negative connotation.

Ability grouping: smalls, informal groups formed within a single classroom. Assignment to an ability group is often short-term, and is not recorded on a student's record.

Detracking: when students are deliberately positioned into classes of mixed ability. Proponents for detracking believe that low-track students will greatly benefit in school achievement if they are mixed in with high-track students

Pages 455 through 456 redacted for the following reasons: S13

#### Key Readings/Scholars

Oakes, Jeannie. Numerous articles and books on 'multiple pathways.'

National Education Policy Centre. April 2011. "Linking Learning to the 21st Century: Preparing all students for college, carer, and civic participation."

Stern, D. and Stearns, R. (2008). "Evidence and Challenges: Will Multiple Pathway Improve Student Outcomes?"

Pastor, M., Brand, B., Mehan, H., Grubb, W.N., Rose, M., Venezia, A., Wheelock, A., Olsen, S., Mason, R., and many more

Harvard: Pathways to Prosperity

### B) Optional topics

Another model of delivery that the 8-10 team presented was to have some of the content within the grade level as optional. On the draft version, the optional content is marked with an asterisk. Allowing some of the content to be optional provides teachers with a bit of the choice and flexibility that the Core/Choice model intended to provide, but that would result in less of a dramatic change in the school structure.

Teachers (and/or students themselves) would choose additional, optional topics based on their students' needs. These needs may include providing the optional content for personal interest and/or to support or extend the required learning.

Further exploration is needed to see if there is a place for optional topics in the elaborations or the layers behind the content.

My notes:

How do we integrate First Peoples' Principles of Learning into math, in a meaningful way, which enhances the learning for not only our FP students, but also for all of our students?

Is the need greater in the content or the teaching? What falls under our realm of influence?

Program evaluation of Math 030 (ethno-math) Catherine McGregor PhD (prov) Peter MacMillan, PhD May 21, 2004

Not Responsive

From:	Marc Garneau	
To:	Colgate, Elaine EDUC:EX; Horner, Melissa EDUC:EX	
Cc:	Ncull@pacificacademy.net; wendy.klassen@ubc.ca; jbarker@sd38.bc.ca; Dawn Driver;   Duncan.fraser@collingwood.org; president@bcamt.ca; \$22	
	S22	
Subject:	Manitoba #Math #Education – Stay the Course   Joy of Education	
Date:	Thursday, April 11, 2013 9:05:34 PM	

An interesting read...

http://joyofeducation.wordpress.com/2013/04/08/manitoba-math-education-stay-the-course/

Pages 460 through 464 redacted for the following reasons: S3 Hi Melissa,

Here is an interesting article regarding research in problem solving and open ended mathematics teaching and lower socio-economic students. I often hear how students in inner city "need" more structure! Great to see you again and looking forward to our work together

Pat

Pages 466 through 516 redacted for the following reasons: S3

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